Third Graders' Mathematics Achievement and Representation Preference Using Virtual and Physical Manipulatives for Adding Fractions and Balancing Equations

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DEDICATION

I dedicate this work to my husband, Tim for being supportive throughout the years of my Ph.D. education and for staying up and listening to my ideas while I wrote my dissertation. This is also for my children, Jeremy and our new baby on the way, just because I love them. I would also like to thank my mother in law, who supported me through this process and watched over Jeremy when I needed to conduct research and write. I would like to thank my mom who always valued education and raised me to believe in myself and to reach for the highest degree. Appreciation goes out to my siblings, Grace, Tammy and Johnny for their love and support. I could not have done this without all of my family's help.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

THIRD GRADERS' MATHEMATICS ACHIEVEMENT AND REPRESENTATION PREFERENCE USING VIRTUAL AND PHYSICAL MANIPULATIVES FOR ADDING FRACTIONS AND BALANCING EQUATIONS

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This mixed method study compared mathematics achievement and representation preference in two third grade classrooms. A total of 36 students learned addition of fractions with unlike denominators and balancing equations in algebra, using two different representations: virtual manipulatives and physical manipulatives.

The project occurred during a two-week time frame where students participated in the activities during their regularly scheduled mathematics class sessions. This research employed a within-subject crossover repeated measures design. For the first unit, fraction addition, Group One worked with physical manipulatives called fraction circles while Group Two worked with a virtual fraction applet. For the second unit, balancing linear equations in algebra, Group One worked with the virtual balance scale applet while Group Two worked with the physical manipulative called Hands-On Equations®.

Overall findings revealed that students in the virtual manipulative treatment outperformed students in the physical manipulative treatment as a whole on the mathematics posttests, $t(35) = 3.87$, $p = .00$. An Analysis of Variance and post hoc tests showed that students in the virtual manipulatives fraction treatment performed statistically better than the students who worked with the physical manipulative fraction circles on the fraction posttest. Students who worked with the virtual manipulative algebra balance scale performed slightly better than the students who worked with the physical algebra manipulatives, Hands-On Equations®, but the difference was not statistically significant. After quantitative and qualitative analysis, the researcher concluded that the fraction virtual manipulative environment better supported the learning of the procedures for the formal algorithm than the physical manipulative environment by providing a step-by-step algorithmic process. In addition, the applet's specific and immediate feedback may have contributed to the higher fraction posttest mean. Students' preference for a tool did not depend on whether it was a virtual manipulative or a physical manipulative, but was determined by students' learning experiences with the specific applet, manipulative tool and mathematical concept. This study highlights that certain features of individual virtual manipulatives may have a greater impact on student learning than others, and certain virtual manipulatives applets may impact the learning of some mathematical concepts more than others.

CHAPTER ONE: INTRODUCTION

High quality mathematics instruction depends on students' engagement with meaningful learning tasks. Effective mathematics instruction allows students to connect their informal knowledge and experience to mathematical abstractions. Learning tasks with manipulatives, physical objects used to represent mathematical ideas, facilitate this connection. Educators have supported using manipulatives for mathematics instruction (Burns, 1996), based on theories claiming that children need physical referents to develop abstract mathematics concepts (Piaget, 1952) and research showing advantages from using physical materials (Sowell, 1989). Best practices in mathematics suggest that children need many experiences with physical materials, visual images, and various forms of representation for mathematical learning to occur.

Physical manipulatives like base-ten blocks, geoboards, color tiles, Cuisenaire rods, unifix cubes, and fraction circles have been available for many years. Many textbook companies offer a manipulative kit with each mathematics series in the primary grades. The National Council of Teachers of Mathematics (NCTM) encourages the use of physical manipulatives to introduce concepts and to concretize abstract ideas. Teaching with manipulatives supports the constructivist theory that learning occurs when students construct personal meaning through hands on experiences. Piaget's work (1952) also indicated that students move from concrete, to pictorial, to abstract thinking. Hiebert and colleagues (1997) point out:

> Mathematical tools should be seen as supports for learning. But using tools as supports does not happen automatically. Students must construct meaning for them. This requires more than watching demonstrations; it requires working with tools over extended periods of time, trying them out, and watching what happens. Meaning does not reside in tools; it is constructed by students as they use tools. (p. 10)

The most challenging process when teaching with manipulatives is to facilitate students' ability to transfer what they do with the manipulatives to their conceptual and procedural understanding.

Now, technology offers new teaching and learning tools called "virtual manipulatives" (Moyer, Bolyard, & Spikell, 2002). These new tools might help this translation between physical objects and students' development of procedural and conceptual understanding and may aid in linking mathematical concepts and procedures. Papert (1993) stated, "the fundamental ingredients of educational innovation must be better things to do and better ways to think about oneself doing these things." So one might question how virtual manipulatives are going to be different from physical manipulatives when teaching and learning mathematics? Will this technology give new ways to solve problems and manipulate mathematical objects? This study attempted to answer these questions.

Background of the Problem

In the past, many of the mathematics software technology available for young children included programs that taught computation skills and traditional algorithms and gave feedback on right and wrong answers. It has been widely believed that physical rather than computer based materials were more appropriate for younger children. However, research on young children and technology indicates that the use of technology can be “developmentally appropriate” when used properly (Clement & Nastasi, 1993). Children show comfort and confidence when using software and can follow pictorial directions and use visual cues to understand their activities.

The National Library of Virtual Manipulatives (www.matti.usu.edu), created by Utah State University and funded by the National Science Foundation, includes over 75 virtual manipulatives for grades K-12 with the emphasis on K-6. These interactive computer based manipulatives resemble their physical counterparts commonly used in mathematics instruction and were created to promote student engagement. The creators of the National Library of Virtual Manipulatives designed the virtual manipulatives in hopes that it would add to some of the benefits of using physical manipulatives in the classroom and eliminate some of the drawbacks such as: classroom management, structuring activities with manipulatives, connecting manipulative use with symbolism, and lack of resources and professional development (Dorward, 2002). Dorward further purports that “the virtual manipulative enhances the physical manipulative and increases the accessibility to these kinds of models of mathematical concepts” (Dorward, 2002, p. 330).

General Statement of Problem

The purpose of this project was to compare student achievement and representation preferences when teaching addition of fractions with unlike denominators and balancing linear equations using virtual manipulatives and concrete manipulatives.

The two virtual manipulatives used in this study came from the National Library of Virtual Manipulatives. The virtual manipulative applets are called Adding Fractions and Algebra Balance Scales. The two concrete manipulatives used in the project were fraction circles and the Hands-On Equations® Gear. The researcher wanted to identify any unique features that existed within the two types of manipulatives by comparing advantageous characteristics. One particular feature with high potential for impacting mathematics learning was the linking representation that exists in the fraction and algebra virtual manipulative environments. That is, the moves made on the screen with the manipulatives are linked to a symbolic notation that appears after each move of the computer mouse. This research examined how this linking feature might play a role in connecting procedures and concepts for students. Data were collected that assessed students' understanding using questions with symbolic representations, pictorial representations and word problems to determine the impact of this feature.

The fraction and algebra virtual manipulatives chosen for this project were concept tutorials that included instructions on learning tasks and gave immediate feedback as a feature of the applets. These particular applets provided a link between the actions of the virtual manipulatives with the numeric representation. For example, the fraction applet connected the algorithm of addition of fractions with a dynamic pictorial representation of fraction addition. The algebra applet connected the symbolic notations of algebra as it worked with a balance scale representing the linear equation.

The researcher became particularly interested in examining virtual manipulatives that have the linked representations after reading Kaput's (1992) article, "Technology and Mathematics Education," where he posed several open-ended questions. One question was, "How do different technologies affect the relation between procedural and conceptual knowledge, especially when the exercise of procedural knowledge is supplanted by (rather than supplemented by) machine?" (p. 549). The National Library of Virtual Manipulatives Adding Fractions and Algebra Balance Scale both have linked symbolic representations that correspond to the moves made by the user. These symbolic representations are what Kaput would refer to as the procedural knowledge that is supplanted by the program.

By comparing the use of virtual manipulatives and physical manipulatives to teach fraction and algebra concepts, this study gives direction to other researchers and educators on the use of virtual manipulatives in school mathematics.

Significance of the Problem

Virtual manipulatives are readily available on the Internet and on CD ROMs developed by textbook publishers. However, research on students' achievement using virtual manipulatives is scant. More research is needed on the effectiveness of virtual manipulatives in order for educators to determine the value and advantageous characteristics of these tools. In addition, the effect on students' preferences for learning with these tools requires further study.

The mathematics content of fractions and algebra were chosen because of the challenge they pose for students. Students often have less out-of-school experiences with fractions than with whole numbers, making it necessary for teachers to provide relevant experiences to enhance students' informal understanding of fractions and help connect procedural knowledge with conceptual understanding. Developing visual models for fractions is critical in building understanding for fraction computation. Yet conventional instruction on fraction computation tends to be rule based. Students are not as fluent in operating with rational numbers as they are with whole numbers. For example, on the National Assessment of Educational Progress (NAEP), also known as "the Nation's Report Card", only 50% of 13 year olds correctly completed problems like 3½ - 3 1/3 and 4 x 2½. One conclusion that was drawn from the result was that by age 13 many students have not fully developed procedural fluency (National Research Council, 2001, p. 138).

In recent years, the question of whether or not algebra should be introduced in elementary school has become a topic of debate. Traditionally, algebra has been seen as a subject taught in middle and high school for its abstract nature. However, algebra has become an integral part of the elementary curriculum. The rationale for introducing algebra so early is based on the premise that much of the curriculum dealing with numbers and patterns leads to algebra. Algebra builds on the proficiency that students develop in arithmetic. However, many students see algebra, with its symbolism, equation solving, and emphasis on relationships among quantities, as an entirely new experience from learning arithmetic.

Many times, procedural fluency is emphasized before conceptual understanding is developed. A recent report indicates that, "Rules for manipulating symbols are being memorized but students are not connecting those rules to their conceptual understanding nor are they reasoning about the rules" (National Research Council, 2001, p. 234). As this project assessed students' achievement, it explored whether the manipulative environments facilitated the connection between conceptual and procedural understanding of fraction and algebra concepts.

Research Questions and Hypotheses

This project was particularly interested in answering the following questions:

1) What impact do virtual and physical manipulatives have on students' achievement when adding fractions with unlike denominators and balancing equations in algebra?
    (a) Does the use of virtual or physical manipulatives facilitate the connection between pictorial and symbolic notation or in terms of conceptual and procedural knowledge?
    (b) What unique features exist within the two types of manipulative environments that impact student achievement?
2) What representation preferences exist between the virtual environment and the physical environment in teaching fractions and algebra?

<u>Null Hypothesis</u>: There are no significant differences in the achievement test scores between students grouped by experimental treatments.

Null Hypothesis: There are no significant differences in representation preferences between virtual or physical manipulatives.

The researcher hypothesized that virtual manipulatives would be more effective than physical manipulatives for connecting symbolic notation to work with manipulatives. This was based on the hypothesis that students would be able to translate their work into symbolic notation better when working with virtual manipulatives because of the scaffolding provided by virtual applets and the feature of a linked representation. In addition, having the linked representation might lessen the cognitive load on the students as they worked with the virtual manipulatives to translate from concrete to symbolic representations. Based on these premises, students would score higher on the assessment items involving fraction and algebra symbolic notations and would score higher in their overall achievement.

Different features provided by the virtual and physical environments would allow distinct opportunities for support and constraints on students' learning experiences. The researcher hypothesized that students would prefer the virtual or physical manipulative based upon their opinions and experiences with the tool that helped them to learn the best.

Definition of Terms

For the purpose of clarity, I define key terms used throughout this dissertation.

1) Physical manipulatives are objects to be handled and arranged by students and teachers that are used to convey abstract ideas or concepts by modeling or representing their ideas concretely (NCTM, 2000). Manipulatives include an array of items such as tangrams, number cubes, 3-D models, and fraction circles.
2) Virtual manipulatives have been defined as follows:
   - Computer based renditions of common mathematics manipulatives and tools (Dorward, 2002, p. 329);
   - An interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge (Moyer, Bolyard & Spikell, 2002, p. 373);
   - Computer software which emulates physical manipulatives by keyboard operation instead of physical action on three dimensional objects (Kim , 1993; Terry, 1996); and
   - Interactive concept tutorials mostly in the form of Java applets or activities. (Drickey, 2000, p. 4)

3) Microworlds are types of virtual manipulatives environments that offer learners open worlds in which they can freely explore problem situations.

4) Tutoring systems are another form of virtual manipulative environment that provides students with strong guiding feedback and linked representations usually in the form of symbolic notation.

5) An applet is a smaller version of an application program. Virtual manipulatives are often placed on the web as applets because each one can act as a stand-alone program.

6) Linked representation is a feature of some computer applets that allows symbolic notation to appear on the screen linked to the actions of the virtual manipulatives.

7) An affordance is the design aspect of an object that suggests how the object should be used. For example, the affordance of the virtual base ten blocks is that one can break apart and glue together groups of tens and hundreds. This unique affordance provides a model for decomposing and composing numbers.

CHAPTER TWO: REVIEW OF LITERATURE

Introduction

The purpose of this chapter is to present a summary of the literature related to all aspects of this research project. First, the chapter begins with an overview and synthesis of existing research on physical manipulatives. Second, research on technology implementation in schools and computer-manipulated programs is summarized since they preceded virtual manipulative technology. The third section on virtual manipulatives and computer-based manipulatives has very recent research, since the technology is still quite new to education. This section begins with the definitions of two types of virtual manipulatives: microworlds and concept tutorials. Then it discusses the unique feature of linked representations. It highlights current research that explores the effectiveness of virtual manipulatives. The fourth section discusses the literature on different modes of representation focusing on the importance of translations between and among pictorial, symbolic, manipulative, verbal, and real life representations. The fifth section includes a discussion on procedural fluency and conceptual understanding. This research focuses on how the translation between manipulative and symbolic representations affects procedural and conceptual understanding. Finally, based on the information presented in this literature review, the last section provides the rationale for the proposed research project.

Research on Physical Manipulatives

Physical manipulatives have been around since the beginning of time. Some examples of historic manipulatives are counting beads, the abacus and counting sticks. Physical manipulatives are objects that can be handled and arranged to stimulate understanding of abstract mathematical ideas. Some examples of commercially available manipulatives are tangrams, unifix cubes, base ten blocks, Cuisenaire rods, geoboards and color tiles. Educators support the use of physical manipulatives based on the tenets of constructivism and other experiential learning theories that state 1) learning is based on experience, 2) sensory learning is the foundation of all experience, 3) learning is the process of proceeding from the concrete to the abstract, and 4) learning requires active participation by the learner (Reys, 1971).

Suydam and Higgins (1977) published a comprehensive review of research conducted in grades K-8 on the uses of physical manipulatives. They found that students using manipulatives demonstrated greater achievement than those not using them. The key to their findings was that physical manipulatives would yield positive results if the "manipulative was used well"(p. 92). In another meta-analysis of research studies done by Parham (1982), there was a difference in achievement scores, with students who had used physical manipulatives scoring at about 85% on the California Achievement test, as opposed to students not using the physical manipulatives scoring at the 50th percentile. Sowell (1989) conducted a comprehensive meta analysis of 60 studies conducted from 1953 to 1987 on the effectiveness of using physical manipulatives for students in grades kindergarten through college. Results showed that mathematics achievement increased through long-term use of concrete instructional materials and that students' attitudes toward mathematics improved when they had instruction with concrete materials provided by teachers knowledgeable about their use. The key to Sowell's analysis was that length of treatment using physical manipulatives was related to achievement.

From 1979 to 1983, researchers from three universities worked on a project called the Rational Number Project (Behr, Lesh, Post & Silver, 1983). Their main focus was to explore the role of physical models as facilitators of the acquisition and use of mathematical concepts as learners move from concrete to abstract. The Rational Number Project research did not identify the "best" manipulative aid for illustrating (all) rational-number concepts but rather recognized that different materials were useful for modeling different real-world situations or different rational-number subconstructs (i.e., part-whole fractions, ratios, operators, proportions). Researchers used concrete manipulatives like fraction circles, cuisenaire rods and even paper folding to represent fraction concepts. The goal was to identify manipulative activities using concrete materials whose structure fit the structure of the particular rational-number concept being taught. Analysis showed that physical models were only one component in the development of formal representational systems, and that verbal, pictorial, and symbolic modes of representation also played a role in the acquisition and the use of concepts (Lesh, Landau, & Hamilton, 1980). They suggested that the ability to make translations among and between these modes of representation was what made mathematical ideas meaningful to learners.

Balka (1993) described the benefit of using manipulatives by stating:

> The use of manipulatives allows students to make the important linkages between conceptual and procedural knowledge, to recognize relationships among different areas of mathematics, to see mathematics as an integrated whole, to explore problems using physical models, and to relate procedures in an equivalent representation. (p. 22)

Despite the support for the use of physical manipulatives in the classrooms, they are not widely used. Kim (1993) cited some of the challenges of implementation: 1) classroom management; 2) structuring, monitoring, and assessing the use of manipulatives; 3) relating manipulatives to mathematical symbols and procedures; 4) lack of financial resources; and 5) lack of professional development.

Ball (1992) also warned against the "magical hopes" that many teachers have about manipulatives. She wrote, "Manipulatives and the underlying notion that understanding comes through the finger tips have become part of educational dogma" (p. 17). She noted several examples of the mere use of manipulatives failing to deliver understanding, and concluded that manipulatives cannot be used effectively without better understanding of how children learn and without adequate teacher preparation. For example, Moyer (2001) reported on how ten middle school teachers used manipulatives to teach mathematics concepts in a year long project. Through interviews and observations, Moyer explored how and why the teachers used the manipulatives in their classrooms and showed that teachers did not always understand the purpose of using the manipulatives and did not use manipulatives effectively when they were not able to represent mathematics concepts themselves.

Baroody's (1989) article "Manipulatives Don't Come with Guarantees" reiterates Ball's point that manipulatives and other tools are not sufficient on their own. He believes that teachers should guide students in building understanding with manipulatives and other tools with meaningful representation of mathematical concepts.

Some of the objections to the use of manipulatives stem from the fact that manipulatives can make previously difficult work appear easy and can mask a lack of understanding. Ball (1992) gave the example of students carrying out a subtraction correctly with manipulatives by following rules they had memorized. However, when students were not given the manipulatives, they reverted to their previous mistakes. Despite these pitfalls, Ball acknowledged that manipulatives have an important role to play by enhancing the modes of learning and communication available to students. She emphasized that manipulatives do not cause the mistake. If a student can do multiplication correctly with manipulatives, but makes a mistake when working with the symbols, teacher guidance and analysis of the error can correct these misconceptions. She reported that manipulatives can provide the scaffolding needed until students can build a formal understanding of the concepts.

Kaput (1989) discussed that one problem with using manipulatives is that sometimes the connection between the actions on the manipulatives and the actions on the symbolic notation are unclear. The problem is that the cognitive load that is imposed during the activity with the manipulatives is too great for students. In other words, students have to keep track of too many procedures in their head and fail to see the connection between their manipulation with the blocks to the symbolic notations.

Based on this research, it is clear that deliberate attention must be paid to help students transfer what they know in the context of the manipulatives to other representations, including symbols, numbers, and graphs. Transfer does not just happen spontaneously. The goal of establishing intentional learning can be accomplished by utilizing a well-designed task sheet and allowing children to reflect on the task through class discussions. Another way to help students reflect is by having guided inquiry while using the medium. Ball (1992) discusses the importance of pedagogy when using manipulatives and how it should be based on the guiding principles of constructivism. There must be an emphasis on personal construction of meaning in every manipulative activity. Communicating about mathematics goes hand in hand with the new emphasis on developing students' reasoning abilities. Classroom discourse about mathematics helps students clarify their thinking and helps teachers observe students' thought processes. When students talk about mathematics, they internalize concepts and build long-term knowledge. In addition, teachers should encourage their students to write about mathematics. One primary purpose of manipulatives is to offer an opportunity for discussion among students, and for discussion between student and teacher.

In an article entitled “Rethinking Concrete Manipulatives,” Clement and McMillan (1996) delineate two major problems with research on physical manipulatives. The first problem is that there is an assumption that the sensory concrete materials hold the mathematical concepts and that children see the same picture that teachers see when using the physical manipulatives. Holt (1982) reports that he and his teachers were excited to use the base ten rods because they saw the link between the rods and the number system. However, they found out that children who don’t already know the concept could not automatically associate the physical manipulative to the concept, and children who already understood numbers could perform the tasks with or without the blocks. He concluded that, “Children who could not do these problems without the blocks didn’t have a clue about how to do them with the blocks. They found the blocks as abstract and disconnected from reality; mysterious, arbitrary and capricious as the numbers that these blocks were supposed to bring to life” (Holt, 1982, p. 219). Another problem is that many times physical actions with certain manipulatives suggest a different mental action from what the teacher wishes students to learn. For example when adding 5 + 4, students using the number line counted on from 5 pointing to the dots, one, two, three, four, instead of using the intended count-on strategy like starting at 5 and saying six, seven, eight, and nine.

Meira’s (1998) research examined how children make sense of physical devices designed by experts to foster mathematical learning and how the use of such devices enables learners to access mathematical concepts. Meira found that instructional tools are only meaningful and “transparent” when used with meaningful learning activities. The researcher concluded that pedagogical tools, such as physical devices or computer microworlds, should be used in the classroom, but they don’t have intrinsic value in and of themselves. The researcher recommended that teachers focus on how students actually use instructional devices in activity and on the transformations of mathematical thinking that take place while they use them. Students must have opportunities for mathematical discussion in the context of which the instructional materials become the tools for talking and writing about mathematical ideas.

Clement and McMillan (1996) claim that the essence of the problem is the misuse of the word concrete. They define two types of concrete knowledge, which they call sensory-concrete knowledge, and integrated-concrete knowledge. The sensory concrete knowledge is when students use sensory materials to make sense of an idea like using counters to add or subtract. For this early stage, children cannot count or add meaningfully without having actual objects to touch and manipulate. The integrated concrete knowledge is the interconnected structure of knowledge like when a child thinks of a fraction problem, such as ¾ + ¾ =, as a money problem, such as $.75 + $.75 = $1.50, which is 1½. When children have this type of interconnected knowledge, the physical objects, the actions they perform on the objects, and the abstractions are all interrelated in a strong mental structure. In other words, the ideas become as tangible and as concrete as the material itself and become a tool for understanding. Mathematical ideas are ultimately made integrated-concrete not by their physical or real world characteristic but rather by how "meaningfully" they are connected to other ideas and situations (Clement & McMillan, 1996, p. 2).

Summary of Research on Physical Manipulatives

Despite all the public opinion and research that promotes the use of physical manipulatives, there are still many challenges teachers face in using physical manipulatives. The review of literature also reveals some confounding results from research that blur the overall effectiveness of physical manipulatives. One issue that has surfaced from the literature is that physical manipulatives can affect achievement if they are used long term and appropriately by teachers who are knowledgeable about their use.

Another important idea that emerged from reviewing literature on physical manipulatives is based on Clement and McMillan's redefining of the word 'concrete' Students do not necessarily need the aid of physical objects for them to build integrated concrete understanding. Virtual manipulatives might support integrated concrete experiences even though these representations are viewed on a computer screen. According to Kamii and Dominick (1998), good concrete activity is good mental activity, which suggests that computer environments can offer just as meaningful of a learning experience as physical manipulatives. The next two sections will discuss how computer environments can enhance mathematical learning.

Research on Technology and Computer Manipulated Programs

Research on the use of technology and computer manipulated programs in mathematics is essential in understanding the development of virtual manipulatives. This section illustrates how technology's role in schools has evolved through the years, from being a didactic tool to becoming an interactive tool. It also makes a distinction between computer manipulated programs and virtual manipulatives.

Technology Integration in Schools

In the past, many of the available software programs were computer-assisted instructions (CAI), which were mostly drill and practice. Olds, Schwartz, and Willie (1980) classified traditional ways that the computer was being used in the eighties. The first classification of computer use was through computer games. Within this category, they separated games into two types: content games, which taught some subject matter, and process games, which involved more problem solving applications. The second category was computer tutorials which simulated a tutor-tutee relationship and where many of the earliest mathematics tutorials were designed to guide students through learning a standard algorithm. The third classification involved computer simulations and microworlds. An example of this is a computer pendulum that students use to simulate friction and motion. The fourth classification grouped specialized and general computer tools like databases that managed airline reservations. The fifth classification was computers as tool-makers. Some examples were the Hypercard and Authorware softwares that allowed users to program animation or other tools.

As more schools became technology rich environments, the role of computers changed. Soon, the emphasis shifted from students learning didactically from computers to constructing knowledge by interacting with technology. The use of new technologies became closely associated with the learning theory of constructivism. Effective use of computers meant student centered learning tasks that allowed for meaningful knowledge construction. With the availability of the World Wide Web, students gained access to real data and worked on authentic problems. Jonassen (1991) described these tasks as having "real-world relevance and utility, that integrated those tasks across the curriculum, and provided appropriate levels of difficulty or involvement" (p. 29). In Jonassen’s (1998) article, “Computers as Mind Tools for Engaging Learners in Critical Thinking,” he defined mind tools as computer applications that engaged learners in critical thinking about the content they were studying. Two of the mind tools he described were the microworlds, Geometry Supposer and Algebra Supposer, that were standard tools used for testing conjectures and manipulating geometric and algebraic objects.

When the National Council of Teachers of Mathematics published the *Principles and Standards for School Mathematics* (2000), technology became one of the six major principles of school mathematics. The principle stated, “Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances student learning” (p. 24). According to this document, technology supports effective mathematics teaching when teachers create appropriate mathematics tasks that capitalize on the strengths of technology, which are the ability to graph, visualize, simulate and compute. In addition, it states that when students work with virtual manipulatives, students can extend their physical experiences and develop more sophisticated mathematics understanding. Some examples that were given were the use of dynamic geometry software to experiment with geometric concepts and the use of graphs to explore characteristics of functions in algebra.

Research reports that computers are used more often in mathematics than in any other school subject (Kober, 1992). Fey (1989) outlined some revealing research about technology in mathematics. He pointed out that across grade levels and ability groups, students who use calculators and computers in mathematics showed improved attitudes and confidence in their mathematical abilities. In addition, students who used technology to solve problems showed more persistence and effort and were more willing to take risks. He also stated that one of the advantages of computer technology was the unique potential to visually represent abstract mathematical ideas.

Bitter, Hatfield and Edward (1998) stated that the use of technology for mathematics instruction enhances “mathematical thinking, student and teacher discourse, and higher order thinking by providing the tools for exploration and discovery” (p. 39). They list ten characteristics of using computer tools to enhance learning.

1. Promotes active versus passive learning.
2. Offers models or examples of exemplary and nonexemplary instruction.
3. Is illustrative and interactive.
4. Facilitates the development of decision making and problem solving.
5. Provides user control and multiple pathways for accessing information.
6. Provides motivation and allows for variability of learning styles.
7. Facilitates the development of perceptual and interpretational abilities.
8. Offers efficient management of time for learning and less instructional training time.
9. Allows for numerous data types.
10. Offers multilingual presentation. (p. 106)

Clement and Sarama (2002) reported that technology offered unique opportunities for learning through exploration, creative problem solving, and self-guided instruction. Software that was drill and practice led to gains in specific rote skills but did not promote mathematical thinking. Open-ended projects and problem solving tasks kept students engaged longer and allowed students to actively search for diverse solution paths.

Computer Manipulated Programs

There are two types of virtual representations on the Internet that are often mistaken for virtual manipulatives. In order to define virtual manipulatives, Moyer, Bolyard and Spikell (2002) made a distinction between virtual manipulatives and virtual mathematics activities. According to these authors, the virtual mathematics activities include only static images or representations. These images do not allow for interactivity since they are static images or representations on the screen, which the users cannot manipulate. One example is the site Visualizing Fractions at www.visualfractions.com.

Another type of virtual mathematics activity is called computer-manipulated images or representations. Computer manipulated representations show representations of manipulatives but do not allow the user to move the images. Instead, the computer moves the images based on a command by the user like clicking a button or typing in a number. They offer an example where the user might type in a multiplication problem and the computer displays a rectangular array. Another example of a computer manipulated representation was described by Essex, Lambdin, and McGraw (2002) in an article called " Racing Against Time." This study looked at a computer program called Trips, which analyzed the rate of change and allowed students to simulate an event where two runners move from a house to a tree. The user is able to see a table and graph as they change the speed over the distance and time.

Anderson, Boyle, and Reiser (1985) describe the Geometry Tutor as a prototypical example of an intelligent tutoring system. It guides learners in the construction of a mathematical proof in geometry, providing immediate feedback, clear hints and help when the learner fails or gets lost, but it accepts only those learners' explorations that are likely to lead to a correct proof. One of the drawbacks of these virtual representations is that they lack the interactivity, which diminishes their potential for meaning construction. For example, in the case of tutoring systems, the close interaction with the computerized tutor can guarantee certain performances, but does not determine the nature of the underlying learning. The problem is that the tutor feedback focuses more on the specific lesson than on how the learner's knowledge is constructed. Another drawback is that learning in such an environment could mean learning how to obtain the best hints and help from the tutor so that the problem at hand can be solved. In other words, the student can learn how to optimize the use of the tutor feedback system instead of the knowledge the task is supposed to convey.

Summary of Research on Technology

Technology is altering the way that children learn mathematics and the way that teachers teach mathematics. As technology becomes more integrated into the curriculum, instructional technology designers need to create the most effective and meaningful instructional tools. Constructivism has been the prevailing philosophy influencing the development of instructional technology. These developments and philosophical considerations were all important aspects in the development of virtual manipulatives.

Research on Virtual Manipulatives

Search Procedures

The literature search for virtual manipualtives produced very recent research because this technology is relatively new. In fact, many teachers still do not know that virtual manipulatives exist on the World Wide Web. Relevant research was located with the help of computer assisted search engines like the Educational Resource Information Center (ERIC), PsycInfo, Expanded Academic Index, Dissertation Abstracts and www.proquest.com that offered digital dissertations. The two main descriptors used in the search were 'virtual manipulatives' and 'computer based manipulatives.' Studies of computer based manipulatives were included because many were virtual manipulatives on CD Roms. Many of the articles were found in recent issues of educational journals like the *Journal of Research in Mathematics Education, Educational Studies in Mathematics, Journal of Special Education Technology, Teaching Children Mathematics,* and *Arithmetic Teacher*. Some were conference papers, or papers from proceedings. Dissertations also contributed background and relevant ideas, although the search only yielded three dissertations on work completed with the National Library of Virtual Manipulatives.

Defining Virtual Manipulatives

What are virtual manipulatives? Virtual manipulatives have been defined as "computer based renditions of common mathematics manipulatives and tools" (Dorward, 2002, p.329). Moyer, Bolyard, and Spikell (2002) defined a virtual manipulative as "an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (p. 373). In fact, according to these authors, for a visual representation to be considered truly a virtual manipulative, one must be able to "slide, flip, and turn the dynamic visual representations as if it were a three dimensional object" (Moyer, Bolyard & Spikell, 2002, p.373).

Difference between Microworlds and Tutoring Systems

There are two distinct types of virtual manipulatives. These systems stand at two extreme ends of a continuum of computer-based learning environments. One is called microworlds and the other tutoring systems. Microworlds offer learners open worlds in which they can freely explore problem situations. On the other hand, tutoring systems provide students with strong guided feedback. But in both cases merely interacting with the machine is insufficient.

According to Thompson (1985), “a prime reason for using a computer microworld is that they form a ‘playground’ somewhere between concrete models and abstract formalisms for developing intuitions of abstract concepts” (p. 466). The constraints in the microworld help turn play into mathematical activities. Students are able to pose problems and make conjectures (Clement & McMillen, 1996). Microworlds' free exploration offers a rich range of experiences, but does not guarantee that specific learning will occur since students might not accept or even notice the educator’s agenda (Hoyles & Noss, 1995). One drawback of these learning environments is that the student may focus on screen events not relevant to mathematics learning.

The tutoring systems or concept tutorials are more specific and are designed with a specific mathematics concept in mind. They are interactive in that students work in a reactive environment. That is, if a student moves a piece off of an algebra balance, the computer will say, “You cannot move the x unless there is at least one x on each side.” This constraint allows for students to move through a directed sense-making pathway. Many tutorials have these built-in supports and constraints. The two virtual manipulative applets chosen for this project are tutorial systems that are linked with symbolic notations.

Linked Representations

Kaput (1995) explains that the problem with physical manipulatives is that people cannot keep record of everything. The connections between the actions on the manipulatives such as Base ten blocks and the actions of representation by formal mathematical notation are often masked because the cognitive load imposed during activity with the blocks is actually too great. Virtual manipulatives can use scripts as recording devices and display the symbolic notation simultaneously. Therefore, linked representations can support students as they concretize abstract concepts.

According to Dorward and Heal (1999) there are several benefits to the virtual manipulatives with linked representations:

> While appropriate use of good physical manipulatives has been shown to increase conceptual understanding, these ‘virtual manipulatives’ directly link iconic and symbolic notation, highlight important instructional aspects or features of individual manipulatives, provide links to related web-based resources, and have the potential to record user movements through stored procedures within each application. In addition, virtual manipulatives are very cost effective, versatile, and provide at least as much engagement as physical manipulatives. (p. 1511)

Char (1991) described how students working with a virtual bean stick used numbers that were available as labels. The availability of the linked representation made the connection between the manipulatives and the numbers more explicit than when using physical bean sticks.

Linked representation may solve the problem of the manipulative being separated from the symbols. It can help support students working at different conceptual levels. That is, the linked representation can be used as a scaffold for students who need symbolic notations displayed on the screen while working with the manipulatives.

Kaput states that different media have different “carrying dimensions” that affect the encoding of information (Kaput, 1995, p.523). Clement and McMillan (1996) support Kaput’s statement by explaining how actions, like breaking computer base ten blocks then gluing them together to form tens, can help students build mental actions of composing and decomposing numbers. The numbers represented by the base ten blocks are also dynamically linked to number displays and automatically change based on the users action, which can help students make sense of their activities and the numbers. Computers allow students to make their knowledge explicit, which helps them build integrated-concrete knowledge.

Research on Effectiveness of Virtual Manipulatives

Berlin and White (1986) investigated the effects of combining interactive computer simulations and concrete activities on the development of abstract thinking. They studied second and fourth grade children using computer simulations compared to concrete materials or a combination of simulations and concrete materials. The tasks developed for this study included pegboards and colored cubes and/or computer simulations of these materials. Students were asked to recognize and duplicate designs, recognize and extend patterns, and perform spatial orientation and discrimination tasks. The focus of Berlin and White's work was to show the use of computer simulations as an interactive link between concrete and abstract thinking. A progression of learning steps was suggested beginning with concrete experience, then to semi-concrete representations (pictures, images), and finishing with internalization to abstract thought. Results showed that not all students were influenced in the same manner by the use of computer simulations. Gender differences were observed leading the authors to suggest that boys and girls react differently to the use of computer simulations and manipulatives. Socio-cultural backgrounds also correlated to differences in learning in this study. The final conclusion suggested that further research should focus on the nature of the influence of concrete manipulatives and computer simulations on students with different genders and socio-cultural characteristics.

Ball (1988) demonstrated positive effects in using both virtual and physical manipulatives in combination in five fourth-grade classes. Three classrooms used physical and virtual fraction strip manipulatives, and two classrooms used traditional methods with some physical fraction strips but no computer. First students worked with the physical fraction strips to represent different fractional amounts and combine them together. Then they worked with computer manipulatives with similar fraction strips. A t-test of posttest means revealed significant differences between the achievement of the experimental group and the control group. Ball concluded that the use of the virtual manipulatives was effective in improving students' ability to solve fraction addition problems. Although the study showed the strength in the use of both types of manipulatives, it did not try to delineate the different effects from the physical manipulative and the virtual manipulative.

Perl (1990) found disadvantages to using virtual manipulatives were tied to difficulty in knowing how to use the virtual manipulative applets or software, or problems with the physical operation of the computer and lack of necessary financial resources.

Char's (1991) research found the following benefits to using virtual manipulatives over physical manipulatives:

1) Reduces classroom management difficulties since virtual manipulatives do not need to be distributed, collected and reorganized or replaced when there are lost parts.

2) Reduces difficulties in structuring, monitoring and assessing students works since the tutorial systems can provide directions, feedback and hints.

3) Helps students build conceptual linkages among representations of mathematical constructs.

4) Provides clearer models of mathematical ideas through visual techniques such as dynamic motion.

5) Encourages collaboration among students and enables quick reenaction and review of students work.

Thompson (1992) examined how the use of base ten blocks contributed to students' construction of meaning for decimal numeration and construction of notational methods for operations involving decimal numbers. Twenty fourth-grade students were assigned to two groups to work on addition and subtraction concepts. One group used a blocks microworld in which students employed the numeration notation and observed the corresponding manipulation of the blocks. The second group utilized the physical blocks without specific directions on how to use the blocks to solve the problems. Students using the physical manipulatives were asked to record all actions performed with the blocks and their solutions on paper. Students worked with the teacher who asked questions and explained the activities. Achievement on the pretest and posttest after seven days of instruction showed similar gains in accuracy. However, analysis of students' responses showed that students who worked with the microworld often attempted methods that reflected meaningful use of notations, although sometimes these were inaccurate. The students who used the physical manipulatives did not record their actions as they were instructed but separately computed the answers using symbols. Unlike the students who used the microworld, these students did not connect their actions with the manipulatives to symbolic notation because there was not any deliberate constraint built in to the physical environment to support this process. Thompson's research is revealing because he was able to use a deeper qualitative analysis to explore the difference between the use of physical and virtual manipulatives. Despite the insignificant quantitative results, there was a difference in the quality of learning that showed how students concretized abstract concepts better in the microworld with the symbolic notations than with the physical manipulatives. Thompson concluded that the enhanced performance was due to the lessening of the cognitive load as students made the link between concrete and numeric representations by performing a parallel series of actions simultaneously.

In research comparing the effectiveness of virtual manipulatives to physical manipulatives, Kim (1993) studied 35 kindergarten students assigned to hands-on or on-screen teaching groups learning geometry and arithmetic skills. There was no significant difference in the pretest and posttest scores between the two treatment groups. Despite the results, Kim suggests that there are desirable features in the virtual manipulatives that appeal to teachers and students. For example, students in the virtual manipulatives group were able to complete the lesson faster than those using the physical manipulatives.

Terry (1996) studied 102 students in grades two through five using base ten blocks or attribute blocks. This study used a combination of physical and virtual manipulatives and found that students using both manipulatives made significant gains from pretest to posttest as compared with students using either type of manipulative alone.

Clement (2002) suggests that computer manipulated programs can model the mental actions that educators want students to learn better than physical manipulatives. For example, whereas physical base ten blocks must be ‘traded’ (e.g. in subtracting, students may need to trade 1 ten for 10 ones), students can break a computer base-ten block into 10 ones using features of the applet. This ability to break tens and glue groups of tens models the concept of decomposing and composing numbers. The computer also links the blocks to the symbols. For example, the number represented by the base-ten blocks is dynamically linked to the students’ actions on the blocks, so that when the student changes the blocks the number displayed is automatically changed as well. This process can help students make sense of their activity and the numbers.

Clement and Saramas (2002) pointed out that one of the advantages of using computer manipulatives is the ability to save and retrieve work, which allows for students to work on long term projects or continue the work after each class. They observed a kindergartner working with physical pattern block manipulatives, making hexagons by trial and error, but not responding to questions about his strategies. However, when working with the computer manipulative, he seemed more aware of his actions. When he was asked how many times he turned a particular piece to make a hexagon, he was able to answer. They concluded that the computer helped the student to be "more deliberative and reflective" (p. 343). This statement is important to this research because my working hypothesis is that the virtual manipulatives will aid in connecting procedural and conceptual learning through deliberative and reflective acts with the virtual manipulatives and the symbolic notations.

Moyer and Niezgoda (2003) conducted a three-day action research project on patterning with eighteen kindergartners in an ethnically diverse classroom using virtual and physical pattern blocks. During the first session, students worked with wooden pattern blocks. The second day, they used virtual pattern blocks. On the third day, children drew their own patterns on a strip of paper. During each class, students were observed by five observers who recorded anecdotal notes and asked questions throughout the lessons. They also gathered children's work samples from each session and analyzed the patterns according to several criterias: total number of patterns created, number of elements in each pattern stem, number of elements used during patterning, and average number of elements used in each pattern. Analysis of students' work showed that children made a greater number of patterns using the virtual pattern blocks than they did when they drew patterns or used wooden pattern blocks. They also found that students used more elements in their pattern stems when using the virtual pattern blocks than during other sessions. The researchers and observers reported that patterns done with the virtual manipulatives contained twice a many creative elements as patterns done with the wooden blocks.

Reimer and Moyer (2005) reported on action research in a third grade classroom using virtual manipulatives to learn about fractions. Reimer taught 19 third grade students for two weeks using several interactive virtual fraction manipulatives. Task sheets were provided to students on each day that they worked with the virtual manipulatives in the computer lab. Data were collected from pretests and posttests of students' conceptual knowledge and procedural computation, student interviews and attitude surveys. The results indicated a statistically significant improvement in students' conceptual knowledge and a significant positive relationship between students' scores on the posttests of conceptual and procedural knowledge. Student attitude surveys indicated that the virtual manipulatives helped them learn by providing immediate and specific feedback, being faster to use than paper and pencil methods, and enhancing students enjoyment while learning fractions.

As a pilot project to this dissertation research, Suh, Moyer and Heo (2004) conducted a classroom teaching experiment in three fifth-grade mathematics classrooms with students of different achievement levels. Virtual fraction manipulative concept tutorials were used in three one-hour class sessions to investigate the learning characteristics afforded by these technology tools. The virtual fraction manipulative concept tutorials exhibited the following characteristics that supported students during their learning of equivalence and fraction addition:

1. Allowed discovery learning through experimentation and hypothesis testing;
2. Encouraged students to see mathematical relationships;
3. Connected iconic and symbolic modes of representation explicitly; and
4. Prevented common error patterns in fraction addition.

Moyer, Niezgoda and Stanley (2005) focused on second graders' ability to use virtual base-ten blocks to learn regrouping. The study examined how students' interactions with the virtual base-ten blocks would impact their ability to create a pictorial representation of the addition regrouping process. Researchers reported that the use of the virtual applet supported students' conceptual understanding of the addition process. They also found that the virtual manipulatives and the drawings helped second language students verbalize their understanding of the regrouping procedure.

Moyer, Suh, and Heo (2005) examined the impact of using virtual manipulatives with students in different achievement groups and compared this with the use of concrete manipulatives in teaching fractions. The participants were four groups of fifth grade students of high, average and low achievement. Results indicated that there was a significant difference in students' pretest and posttest achievement scores between low achievement and high achievement groups, with students in the low achievement group showing significant improvement compared to the high achievement group. Researchers concluded that the students' achievement levels impacted whether or not the treatment was powerful enough to effect students' test scores. The study suggests that improvement in test scores may be tied to student practice with visual representations of fractions, which may enhance students' abilities to explain and represent their thinking using pictorial representations.

Additional research on virtual manipulatives can be found in recent dissertations that have explored virtual manipulatives from the collection of the National Library of Virtual Manipulatives. Drickey's (2000) dissertation compared the effectiveness of using physical manipulatives against virtual manipulatives to teach visualization and spatial reasoning in the middle school. Drickey was a student of the designers of the Utah State University's National Library of Virtual Manipulatives. Her study involved two treatment groups: virtual and physical manipulatives and one control group where students were taught in a traditional setting with teacher led discussions without any manipulatives. Drickey's investigation focused on finding differences between ability groups and attitudes. Students' achievement scores on the visualization and spatial reasoning pretest and posttest and the math attitudinal scores were compared. In contrast to Moyer, Suh and Heo (2005), results did not show any significant differences on the mean posttest scores among the three groups, by ability or attitudes. The virtual and physical manipulatives groups did report a preference to using manipulatives during instruction. Students in the virtual group did have a higher rate of on-task behavior than the physical and the no manipulative group. The researcher recommended further study involving longer treatment, manipulative use during assessment, larger sample size of differing abilities, and gender differences in achievement and attitude.

Takahashi (2002) investigated the affordances of computer-based manipulatives compared with physical manipulatives. The research involved two sixth-grade classes where one class used a computer-based geoboard, and the other class used a physical geoboard. The two observers and the researcher, who taught all classes, used the Japanese Lesson Study model and discussed their findings to identify affordances of the two types of geoboards in problem-solving activities. The lessons were student-centered and focused on allowing students to discover a formula for the area of a parallelogram. Results revealed that the computer-based and physical geoboards had different affordances. For example, the capability of putting color inside the geoboard shapes was an affordance of the computer-based geoboard. An affordance of the physical geoboard was that it was easy for students to make a shape directly on the geoboard. This study suggests that the computer-based geoboard is an appropriate tool for a class designed to develop a formula for finding the area of a parallelogram by transforming a shape. The physical geoboard, on the other hand, has the potential to be a useful tool to help students develop the concept of area and learn ways to find the area of a rectangle and a square. Therefore, in order to maximize students' learning in problem solving, the researcher recommended that these two types of geoboards be used in complementary roles in the classroom. Takahashi suggests that more research needs to be done to identify instructional activities that take advantage of the affordances of both kinds of geoboards.

Izydorczak (2003) studied the collection of virtual manipulatives from Utah State University's National Library of Virtual Manipulative site to evaluate features of virtual manipulatives that facilitate or fail to aid in mathematical learning. The purpose of her study was to help future development of the applets and contribute to the model of learning with manipulatives. The researcher videotaped weekly sessions with three individual students to learn about the effectiveness of virtual manipulatives in comparison to physical manipulatives. She interpreted the tapes using analytic induction and constant comparison methods. From her study, she concluded that both physical and virtual manipulatives should be used to provide alternate representations for students' to build mathematical concepts. Physical manipulatives were more concrete for students and gave them more access to concepts. Virtual manipulatives were not concrete and led to rote understanding. The researcher recommended the use of situated contexts in virtual environments. The benefits of virtual manipulatives included speed, extensibility and cleaniness. The pitfalls of virtual manipulatives were their inconsistencies, potential to distract, and difficulty with user control. She concluded that the linked representations were not used to their potential, but the constraints built within the virtual environment allowed students to work in the mathematical notation systems. Another conclusion revealed that teacher presence was critical in the virtual environment for providing technical support, assigning tasks appropriate for students' ability, and engaging students' in discussion about mathematical ideas.

Summary of Research on Virtual Manipulatives

The review of literature on virtual manipulatives reveals some confounding results. Similar to the research on physical manipulatives, difficulties with virtual manipulatives can also be experienced when teachers lack training in their use. Many researchers have found that virtual and physical manipulatives used together can enhance mathematical learning, for each environment has unique affordances that provide for different learning opportunities. The general consensus of the literature review on virtual manipulatives is positive and shows that the applets provide an engaging way for students to construct mathematical knowledge. However, there is still a need to conduct more research on virtual manipulatives to learn about their full potential. Dorward (2002) makes an interesting point about reflecting on research and intuition. He asks this question, "When we believe that an instructional innovation has the potential to increase students' achievement and attitude, yet research cannot prove that it is any better than any other method, how can we justify its use? How can we rationalize intuitive decision?" (p. 331). He invites teachers to use the resources at the National Library of Virtual Manipulatives and to conduct action research to reflect on the advantages and disadvantages to contribute to the growing body of research on the use of virtual manipulatives.

Different Modes of Representation

The following discussion of different modes of representation is important to this research because both the virtual and physical manipulatives are forms of mathematical representations. In addition, this project examines the integration of concrete, pictorial and symbolic modes of representation during instruction and assessment. There are several different theories on representation. Most of the literature can be divided into two broad categories including internal and external representations (Goldin & Shteningold, 2001). The external representations are more object-oriented in that they are the representations that can be communicated to others like drawings, graphs or manipulative models. Internal representations are students' mental representations, thoughts, images and encoding of a mathematical idea. The first part of this section discusses research on external models of representation and the second part discusses internal representation and modes of representational thought.

External representations include "student notations and pictures, already-made drawings such as pictures of partitioned objects, and structured materials such as fraction strips and Cuisenaire rods. Structured, in this case, refers to materials that have been designed for instruction of particular mathematical concepts" (Brinker, 1996, p.1). Physical manipulatives, like fraction circles and Hands-On Equations®, are examples of structured materials in the external model of representation classification. Virtual manipulatives, like the fraction and algebra applets, may also be considered structured materials, although they are dynamic images that exist in the virtual environment. The difference, however, is that fraction and algebra applets are virtual and exist in the computer environment and include symbolic representations on the computer screen that correspond and change with the moves made by the user, while the physical materials do not.

Bruner (1966) suggests three modes of representational thought. That is, an individual can think about a particular idea or concept at three different levels. "Enactive" learning involves hands-on or direct experience; sometimes called the concrete level. The next mode of learning that Bruner calls "iconic" is one based on the use of the visual medium like pictures, often called pictorial representation. The last stage is the "symbolic" mode where one uses abstract symbols to represent reality. Bruner (1960) refers to the work of Piaget, stating "what is most important for teaching basic concepts is that the child be helped to pass progressively from concrete thinking to the utilization of more conceptually adequate modes of thought" (p. 38). This theory is important to this research because the assessment of student achievement includes pictorial items, symbolic items, and word problems.

The role of representation in school mathematics is emphasized in the publication of *Principles and Standard for School Mathematics* (NCTM, 2000). It states that instructional programs from prekindergarten through grades 12 should allow students to:

> 1. Create and use representations to organize, record, and communicate mathematical ideas;
> 2. Select, apply, and translate among mathematical representation to solve problems; and
> 3. Use representations to model and interpret physical social and mathematical phenomena. (p. 67)

Reform oriented teachers know that allowing students to create and use representations like manipulatives helps students bridge the gap between physical and symbolic representations. Teachers who use manipulatives effectively bring mathematics to life by presenting students with application problems, i.e., mathematics problems set in real-life contexts. This allows students to move away from the “memorize, follow-the-procedure method” to a more meaningful learning experience.

Lesh, Landau, and Hamilton (1983) identify five distinct types of representation systems that occur in mathematics learning and problem solving and discuss the importance of translation among mathematical representations to solve problems. Figure 1 illustrates these five representation systems which are:

> 1. Real life experience or "scripts", in which knowledge is organized around "real world" events that serve as general contexts for interpreting and solving other kinds of problem situations;
>
> 2. Manipulative models, like Cuisenaire rods, arithmetic blocks, fraction bars, number lines, etc., in which the "elements" in the system have little meaning per se, but the "built in" relationships and operations fit many everyday situations;
>
> 3. Pictures or diagrams, static figural models that, like manipulative models, can be internalized as "images";
>
> 4. Spoken symbols like logical reasoning; and
>
> 5. Written symbols, which, like spoken languages, can involve specialized sentences and phrases (x + 3 = 7) as well as normal English sentences and phrases. (p. 265)

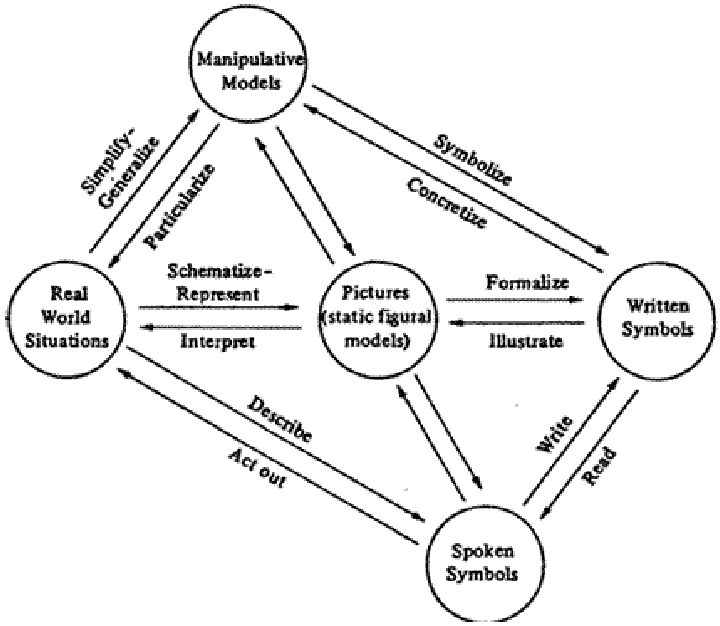


Figure 1. Five distinct types of representation system (Lesh, Landau, & Hamilton, 1983).

These distinct types of representation systems are important, but most important is the translation among the modes of representation, as indicated by the double arrows crossing in the middle of the figure. Translations among the different representations assess whether a student conceptually understands a problem. Some of the ways to demonstrate translation among representations is to ask students to restate a problem in their own words, draw a diagram to illustrate the problem, or act it out.

Cramer (2003) discusses the importance of "representational fluency." She refers to Lesh's translation model and states, "The model suggests that the development of deep understanding of mathematical ideas requires experience in different modes, and experience making connections between and within these modes of representation. A translation requires a reinterpretation of an idea from one mode of representation to another" (p. 1).

Many researchers discuss the importance of translation and exposure to multiple representations. For example, for algebra, Greenes and Findell (1999) state that in order to develop mathematical reasoning in algebra, students need to be able to interpret algebraic equations in various representations like pictorially, graphically or symbolically. They recommend experiences like representing algebraic expressions using the balance scales. In addition, students can build mathematical reasoning by using several different kinds of physical materials to act out given problem situations.

Meyer (2001) recommends realistic mathematics education, which promotes the use of representation in middle school algebra and progresses through levels of abstractions. The first stage of mathematics activity should involve concrete experiences from which abstract ideas can attach meaning. Meyer states that the bridge between concrete and abstract is through students' creation and use of models, drawings, diagrams, tables or symbolic notation. In the instructional sequence, students solve algebraic problems illustrated by balance pans without any equation. This is done on purpose, according to her, so that students will be encouraged to use more informal strategies.

Good problem solvers tend to be flexible in their use of a variety of relevant representational systems and they instinctively switch to the most convenient representation to emphasize any given point in the solution process. Kieren (1980) proposed that in order for students to have a proficient understanding of rational numbers, they need to be exposed to five interconnected subconstructs: part/whole comparison, measures, operators, quotients and ratios. With these multiple representations, students can use their knowledge flexibly and efficiently when solving addition, subtraction, multiplication or division of fraction problems.

Lamon (2001) conducted a longitudinal study examining two urban schools in different parts of the country for four years. There were five classes in grades 3 through 6 that were taught the five fraction subconstructs mentioned above and one control group that was taught using a traditional approach. The groups were not taught any rules or operations. After four years, all five groups developed a deeper understanding of rational numbers than the control group as measured by the number of subconstructs the students were using and by their achievement scores on computation assessments. Lamon concluded that by using different representations of rational numbers, students gained a deeper understanding and were able to transfer their knowledge from one subconstruct to the other.

Summary

This section discussed the importance of the interactions between internal and external representations of problem situations. According to Behr, Lesh, Post and Silver (1983), external representations like pictures, concrete materials, and written symbols, reduce memory load or increase storage capacity, code information in a form that is more manipulable, or simplify complex relationships. The research discussed in this section is important to this study because it analyzes how external representations, including virtual manipulatives with symbolic notations and physical manipulatives, impact students' achievement.

Students' Procedural and Conceptual Understanding

This research examines students' achievement in procedural and conceptual understanding. Often times, procedural fluency and conceptual understanding are seen as competing for attention in school mathematics. For example, trends in mathematics education swing back and forth on a pendulum from teaching the basics in the 1960's, to emphasis on hands-on math in the 70's, then a cry for back-to-basics in the 80's. Finally in 1989 when the National Council of Teachers of Mathematics published its Curriculum and Evaluation Standards, it emphasized building students' mathematical power which it defined as " an individual's ability to explore, conjecture and reason logically as well as the ability to use a variety of mathematical methods effectively to solve nonroutine problems" (NCTM, 1989, p.5). Practice is important, but practice without understanding is a waste of time. Once children understand computational procedures, practice will help them become confident and competent in using them. Research indicates that if children memorize mathematical procedures without understanding, it is difficult for them to go back later and build understanding (Resnick & Omanson, 1987; Wearne & Hiebert 1988).

In an experimental study, fifth grade students who first received instruction on procedures for calculating area and perimeter followed by instruction on understanding those procedures did not perform as well as students who received instruction focused only on understanding (Pesek & Kirshner, 2000). When children memorize without understanding, they may confuse methods or forget steps (Kamii & Dominick, 1998). Children need to learn what computation means and how to do it.

According to the National Research Council, students need to have mathematical proficiency to be successful in mathematics. Mathematics proficiency consists of five strands that are interwoven and interdependent:

1. Conceptual understanding, comprehension of mathematical concepts, operations and relations;

2. Procedural fluency, skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;

3. Strategic competence, ability to formulate, represent and solve mathematical problems;

4. Adaptive reasoning, capacity for logical thought, reflection, explanation, and justifications; and

5. Productive disposition, habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy. (National Research Council, 2001, p.5)

Importance of Conceptual Knowledge

When students have conceptual knowledge, they know more than isolated facts and methods. They are able to represent mathematical situations in different ways and know how different representations can be useful for different purposes. An example related to fractions is a student adding 1/3 and 2/5. They might draw a picture or use physical materials of various kinds to show the addition. They might also show the number sentence as a story problem (National Research Council, 2001, p.119). Conceptual understanding helps students avoid many critical errors in solving problems. For example, one of the common error patterns that Ashlock (2001) identifies in fraction computation is called "adding across." When given a problem like ½ + ¼ =, students who don't have conceptual understanding will answer that as "2/6 " by adding the numerators (1 + 1) and the denominators (2 + 4) without changing the fractions into common denominators.

Lesh, Landau, and Hamilton (1983) state that a student "understands" an idea like "1/3" if: (a) he or she can recognize the idea embedded in a variety of qualitatively different representational systems, (b) he or she can flexibly manipulate the idea within given representational systems, and (c) he or she can accurately translate the idea from one system to another. According to these authors,

> As a student's concept of a given idea evolves, the related underlying transformation/translation networks become more complex; and teachers who are successful at teaching these ideas often do so by simplifying, concretizing, illustrating, and paraphrasing these ideas, and embedding them in familiar situations. (p. 275)

Importance of Procedural Fluency

Procedural fluency is the knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately and efficiently (National Research Council, 2001, p.121). There is no doubt that every child should have an accurate method for computing 34 x 45, ½ + ¾ and 2x + 5 = 15.

However, some children will invent their own procedures for computing, and some children will use more conventional methods they have learned from teachers. Regardless of the source, children's computational procedures need to be both efficient and correct. The development of efficient, correct procedures requires careful instruction that focuses on developing understanding. There is a strong connection between conceptual understanding and procedural fluency. There are some algorithms that are as important as concepts, in that, students need to understand that a particular procedure can be used to solve entire classes of problems, not just individual problems. By learning these algorithms as "general procedures" students can gain insight that mathematics is well structured and filled with patterns.

Learning numerical algorithms is one of the recommendations put forth by the National Research Council (2001), in the book, *Adding It Up*. They define an algorithm as a "reliable step-by-step procedure for solving problems" (p. 414). In the 1998 National Council of Teachers of Mathematics yearbook, *The Teaching and Learning of Algorithms in School Mathematics*, Maurer (1998) defines algorithmic mathematics with two meanings, traditional and contemporary. The traditional meaning emphasizes carrying out the algorithm, and the contemporary meaning, called algorithmics, focuses on developing algorithms, understanding them and choosing among different algorithms to fit the problem. Some algorithms have been discovered by great mathematicians from the past. When children create their own procedures as they work out mathematics problems these are called invented algorithms. Despite the aid of calculators and computers, the need to understand, perform and apply algorithms is essential to building mathematical power and students who can reason mathematically.

Research Implications

This literature search has focused on five areas: 1) background on physical manipulatives, 2) research on technology and computer manipulated programs, 3) literature on virtual manipulatives, 4) theories on different modes of representation, and 5) literature on procedural and conceptual understanding. The review of literature suggests that physical and virtual manipulatives, if used appropriately, can yield many benefits. However, there is insufficient research to determine how virtual manipulatives impact mathematics achievement compared to physical manipulatives. More specifically, how do different manipulative environments enhance the translation between different modes of representation? How do they impact conceptual and procedural understanding of mathematical concepts? Educators have not yet fully realized the potential of virtual manipulatives due to their limited use. The result of this review demonstrates that research is necessary in the area of understanding the impact that virtual manipulatives may have on student achievement and learning preferences. This study contributes to the research and serves to inform future research in the field of mathematical learning with virtual manipulatives.

CHAPTER THREE: METHODS

Research Design

This project occurred during a two-week time frame during regular school hours in a public elementary school. Students participated in the project during their regularly scheduled mathematics class sessions. This research employed a within-subjects crossover repeated measures design (Campbell & Stanley, 1963). All subjects received both treatments, which allowed each student to serve as his or her own comparison. This approach eliminates concerns of individual differences found in between-subjects designs and maximizes statistical power. The one drawback of the crossover design is the potential for distortion due to carryover, that is, residual effects from preceding treatments. To avoid any residual effects, the researcher introduced two completely different mathematics units, fractions and algebra, as the topics of study.

Participants and Setting

The participants in this study were 36 third grade students in two classes at the same elementary school. The student demographics included 83% White, 11% Asian, 3% African American, and 3% Hispanic. There were 22 boys and 14 girls in the project, with 61% male and 39% female. Students at this school were placed in mathematics achievement groups through standardized testing methods. Student chosen for this study were in the middle achievement group working on a third grade level in mathematics.

The school where the project was conducted is approximately an hour from the Washington DC Metro area.

These third grade students had a regular computer lab time scheduled each week for a 45 minute-period where they use word processing applications to create learning projects or used the Internet to research content. They also visited the lab to work on a computer program that taught basic skills in mathematics and language arts. However, these programs were primarily drill and practice.

Each of the two classes was randomly assigned to the virtual manipulative treatment group and the physical manipulative treatment group for the first week of instruction on fractions. During the second unit, on algebra, each group received the opposite condition (See Table 1). Students in both treatment groups were taught by the same teacher, who was also the researcher. Using the same teacher eliminated validity threats due to classroom and teacher differences and helped ensure that both treatment conditions were administered consistently.

Table 1.

Experimental Conditions

	Data collected	Instructional Mode Fraction Unit	Data Collected	Instructional Mode Algebra Unit	Data Collected
Group #1	Fraction & Algebra Pretest	Physical Manipulatives- Fraction Circles	Fraction Posttest and User Survey	Virtual Manipulatives- NLVM algebra applet	Algebra Posttest, User Survey, Interview, and Preference Survey
Group #2	Fraction & Algebra Pretest	Virtual Manipulatives- NLVM fractions applet	Fraction Posttest and User Survey	Physical Manipulatives- Hands-On Equations®	Algebra Posttest, User Survey, Interview, and Preference Survey

Materials

During the virtual manipulative treatments, students used the Internet to work on the website called the National Library of Virtual Manipulatives (http://matti.usu.edu/nlvm/nav/). During the fraction unit, they worked specifically with the “Fraction: Equivalence” and “Fraction Adding” applets in the grade 3-5 Number and Operation section. During the algebra unit, they used the Algebra Balance Scale in the grades 9-12 Algebra section to solve simple linear equations using a pan balance representation. Although the Algebra Balance Scale is designated for students in grades 9-12, it can be used very appropriately in the middle grades. Students in the physical manipulative treatment group used fraction circles and a fraction equivalence mat for the fraction unit and Hands-On Equations® for the algebra unit.

Description of the Fraction Applet

The fraction applets used in this project are found on the National Library of Virtual Manipulatives. These applets are called Fraction-Equivalence, which illustrates the relationship between equivalent fractions and Fraction Adding, which illustrate what it means to find a common denominator and combine two fractions. On the Fraction Equivalence applet, students are shown one fraction circle or Square that they are asked to rename at least three different ways. Using the arrow key students can divide the fraction into multiple parts. On the Fraction Adding applet students are presented with two fraction circles or squares that have different denominators. To find a common denominator, the computer prompts students by asking them to rename the two fractions so that they are the same. To do this, students click on arrow buttons below the whole unit, which changes the number of parts. When they have an equivalent fraction, all lines are red. When a common denominator has been identified, students can type the names of the equivalent fractions into the appropriate boxes. They check their answers by clicking the check button. If they have specified a correct response, the screen takes them to the next step, which allows them to combine the fractions. This can be done in two ways. Students can combine fraction representations by dragging the fraction pieces into a new region called the sum circle or sum square or they can simply type in the answer to the problem and the applet will move the fractional pieces over to the sum circle or square. Each step of the way, the pictures are linked to numeric symbols that dynamically change with moves made by the students (See Figure 2).

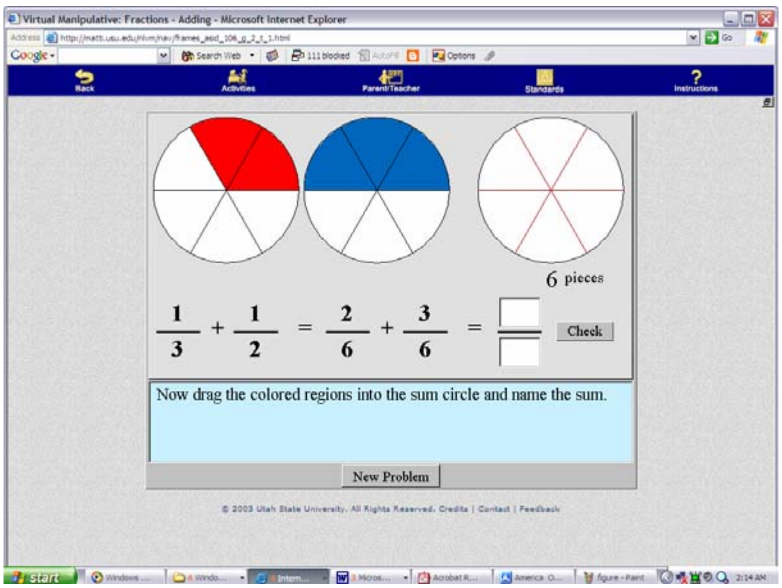


Figure 2. Virtual manipulative fraction adding applet.

Description of the Fraction Circles

The fraction circles used in this project were the Deluxe Fraction Circle Set that consists of nine circles divided into halves, thirds, fourths, fifths, sixths, eighths, tenths, twelfths and 1 whole. The fraction equivalence mat is a learning mat that came from a book called *Fabulous Fractions* published by Activities Integrating Math and Science (AIMS, 2000). The mat has seven fraction circles, which are called Fraction CDs (that stands for common denominators) and are divided into fourths, sixths, eighths, tenths, and twelfths, sixteenth and twenty-fourths (See Figure 4). Students can use the fraction mat to find equivalent fractions or to find common denominators by placing their fraction pieces over the fraction CD's and seeing if the lines line up evenly with their fraction pieces. The instruction on the fraction CDs mat states: " Adding unlike fractions: Example ¼ + ½ = Place ¼ in the circle. Place ½ adjacent to it. Read the outside ring."

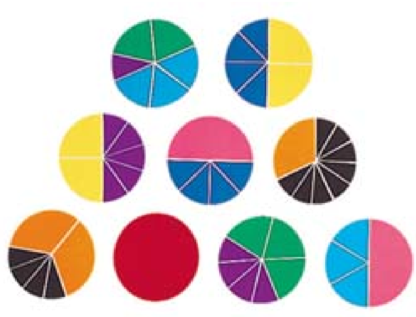


Figure 3. Deluxe Fraction Circles

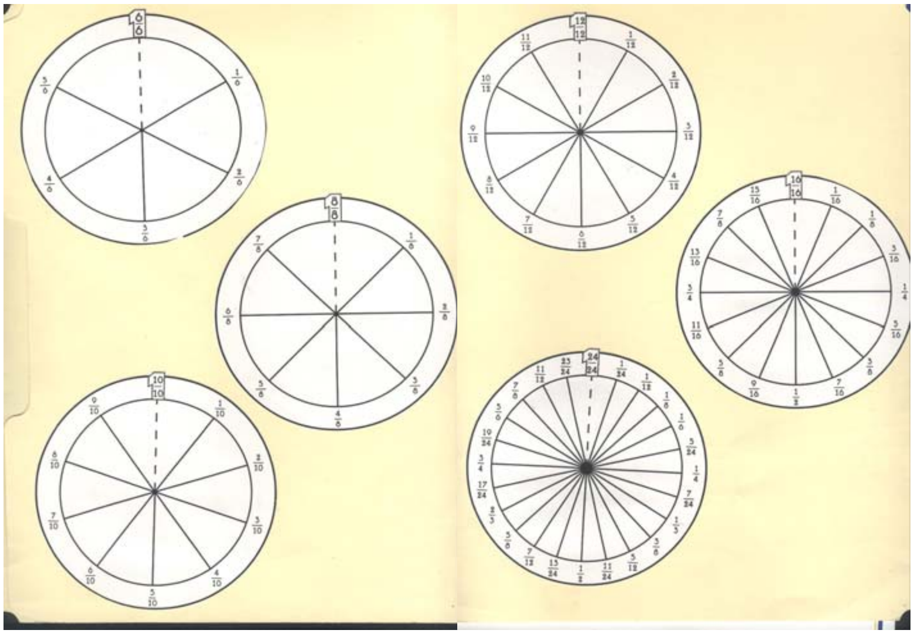


Figure 4. Fraction Mat

Description of the Virtual Balance Scale

This virtual manipulative applet allows students to solve simple linear equations through the use of a balance beam. The red unit blocks, representing 1s and blue x-boxes, representing the unknown x, are placed on the pans of a balance beam. Once the beam balances to represent the given linear equation, students can choose to perform any arithmetic operation, as long as they perform the same operation on both sides of the equation, thus keeping the beam balanced. If the equation is not balanced, the beam will slant to one side. The goal of the applet is to get a single x-box on one side, with however many unit blocks are needed for balance, thus giving the value of x (See Figure 5).

Figure 5. Virtual manipulative Algebra Balance Scales.

Description of the Hands-On Equations®

The Hands-On Equations® materials (Borenson, 1997) are a visual and kinesthetic teaching system for introducing algebraic concepts to students in grades three to eight. The materials were developed by Dr. Henry Borenson. The teacher's edition comes with a stationary plastic balance scale, number cubes and pawn pieces. The student's edition comes with a balance scale mat, number cubes and pawn pieces (See Figure 6). These materials are designed to represent algebraic equations. The pawn pieces represent the unknown x value and the number cubes represent numbers in the equation. The pawn pieces and the number cubes are used with the balance scale mat to model algebraic equations. One of the rules for working with Hands-On Equations® is that when one takes off a pawn or a number from one side, the same number of pawns or number cubes must be removed from the other side of the balance scale to keep the equation balanced.

Figure 6. Hands-On Equations®.

Procedures

Two weeks before instruction began, students took an 18 item mathematics content pretest with fraction and algebra questions. During the first week of instruction, Group One learned fraction concepts using the physical manipulatives in a regular classroom setting. Group Two learned fraction concepts using the virtual manipulatives in the computer lab. The teacher used similar manipulatives in both the virtual manipulative sessions and the physical manipulative sessions. For example, Group One used commercially made fractions circles during the physical manipulative sessions. On each day in the classroom with the group using the physical manipulatives, the teacher modeled several activities for students prior to allowing them to investigate fraction and algebra concepts independently. Group Two used the fraction applets with dynamic images of fraction circles on the computer. Students worked in the computer lab for four days using the virtual manipulatives, with a one-hour lesson on each day. Each day in the computer lab began with an introduction to the virtual manipulative applet that would be used that day and several mathematical tasks for the students. Students were given a teacher-made task sheet that provided instructions for using the virtual manipulatives, several problems, and space to record their work. These directions helped students focus on the mathematical tasks during the lessons. The teacher reviewed the instructions with the class and modeled how to use the virtual manipulatives before students worked independently on the activities. The teacher-researcher lead instruction and discussions with the students during all of the class sessions, both in the classroom and the computer lab. There was a series of four lessons on fraction concepts. On the fifth day, both groups were given the paper and pencil posttest, which they took without the use of any manipulatives (See Table 2).

Instructional Days One through Five

During the fraction unit, the teacher taught fraction concepts that included fraction equivalence and addition of fractions with unlike denominators. The mathematics instruction for the physical and virtual fraction treatment groups was designed to be the same, except for the manipulative environment. The consistency in the lessons and having one teacher teach both classes was important so that there would not be extraneous variability between the two conditions. The only difference between the two conditions was the task sheet. The physical manipulative task sheet included problems written on a paper, whereas, the virtual manipulative group had the problems on the computer screen. Although, the virtual group had a task sheet, it was for students to record their answers and write about what they learned from the lessons (See Appendix D).

The first lesson focused on students using the manipulative tools to find lists of equivalent fractions and constructing a rule from analyzing the patterns of equivalent fractions. The next day, students were introduced to adding fractions with like and unlike denominators. The teacher modeled addition with like denominators then posed a problem with addition of unlike denominators. Students in each treatment were asked to use the physical or virtual manipulatives to model symbolic expressions. For example, students were given a fraction problem and had to model the problem using fraction circles in the physical manipulative treatment. Students were asked to find ways to combine two fractions with unlike denominators by using what they learned the previous day when finding equivalent fractions. They were asked to practice several tasks and then write a procedure that worked for them. Before the end of class, the teacher brought the group together to discuss students' procedures for adding fractions with unlike denominators. These class discussions brought closure to each lesson with guided inquiry. Some questions were 1) Is there a pattern in the list of equivalent fractions? 2) What rule could you make to show how you add fractions with unlike denominators?

Instructional Days Six through Ten

At the beginning of the second week of the study, the treatments were switched so that Group One worked with the virtual manipulatives for algebra called the Virtual Balance Scale and Group Two worked with the physical manipulatives, Hands-On

Equations®, while learning algebra. During the algebra unit, students worked on solving linear equations. The algebra unit lasted for four consecutive days. First, students were shown a simpler model of balancing equation using an arithmetic problem like 2 + 3 = 5 on the balance scale. This was done to show students that the equal sign meant that both sides of the equation were balanced. Then the teacher introduced the idea of x as the unknown. She placed a box with an x written on it over the number 3 and wrote 2 + x = 5 and asked if they could figure out the value of the unknown. After the initial demonstration, students in the physical manipulative treatment group were given a task sheet that had several algebraic equations that they had to model on the balance scale mat and solve. The students in the virtual treatment group were asked to set up the algebraic expression shown on the computer screen on the virtual balance scale and to solve for x. In this process, students learned how to express numbers and the unknown x using objects and pictorial representations. Both groups were asked to keep a record of their mathematical procedures. Class discussion always brought closure to the day's lesson. The teacher asked questions like, 1) What were some strategies you used to find the value of x? 2) How would you describe the rules for finding the value of x to someone who doesn't know algebra? After the four days of instruction, students were given the algebra posttest along with the user and Preference Surveys.

Table 2.

Instructional Sequence

Timeline	Group #1	Group #2
Two weeks before	Pretest on Fraction and Algebra concepts	Pretest on Fraction and Algebra concepts
	Group #1: Physical Manipulatives: Fraction Circles	Group #2: Virtual Manipulatives: Fraction- Equivalence and Adding Applets
Day 1	Introductions to Fractions	Introduction to Fractions
Day 2	Equivalent Fraction	Equivalent Fraction
Day 3	Addition of Fractions	Addition of Fractions
Day 4	Addition of Fractions	Addition of Fractions
Day 5	Fractions Posttest User Survey	Fractions Posttest User Survey
	Group #1: Virtual Manipulatives Virtual Balance Scale	Group #2: Physical Manipulatives Hands-On Equations®,
Day 6	Introduction to Equations	Introduction to Equations
Day 7	Solving Linear Equations	Solving Linear Equations
Day 8	Solving Linear Equations	Solving Linear Equations
Day 9	Solving Linear Equations	Solving Linear Equations
Day 10	Algebra Posttest User Survey Preference Survey	Algebra Posttest User Survey Preference Survey
Day 11	Interviews	Interviews
Day 12	Interviews	Interviews

Data Sources

Several sources of data were collected during project including the pretest and posttest of students' mathematics content knowledge, User Surveys, Preference Surveys, field notes, student interviews, and classroom videotapes. These sources were used to triangulate the data collected during the project.

Quantitative Measures

Pretest. At the beginning of the study, students took a comprehensive pretest that assessed their mathematics content knowledge of fraction addition and algebra prior to the treatments. The purpose of the pretest was to find out the level of prior knowledge students had of adding fractions with unlike denominators and solving linear equations. The pretest had a total of 18 items created by the researcher, nine on addition of fractions with unlike denominators and nine on solving linear equations. The fraction and algebra sections each included four pictorial items, which had pictures and number sentences, four symbolic items, which had only number sentences, and one word problem, which asked students to draw a picture, write a number sentence, and explain their solution. The pretest items were similar to the test items from the posttest to build in reliability between the two tests. The two sections were graded separately to find out prior content knowledge of each mathematical concept: fractions and algebra. Each item on the symbolic and pictorial sections was worth one point and the each word problem was worth two points for a total of 10 possible points for each section of the pretest (See Appendix E).

Posttests. The teacher administered two different posttests of students' mathematics content knowledge: a fraction posttest and an algebra posttest. The researcher created each posttest with 18 items: (a) eight pictorial items, which had pictures and number sentences; (b) eight symbolic items, which only had number sentences; and (c) two word problems, which required students to draw a picture, write a number sentence and explain how they solved it. For the sake of brevity, the researcher refers to the first type of test items simply as pictorial test items even though it had numeric sentences since the key difference between the first two types was that these had pictorial representations, and the second type, the symbolic test items, only had the numeric sentences. The pictorial and symbolic only test questions were worth one point each and each word problem was worth two points for a total of 20 possible points on each posttest (See Appendix F).

The researcher created the posttests with three representational test items: pictorial, numeric and language problems. These items were used to compare achievement among the different representational modes. The fraction pictorial items had pictures of two fractions with unlike denominators with the corresponding number sentence written on the bottom of each. For the fraction numeric items students were given only the following number sentence (2/3 + 1/4 =) without the aid of any pictures. Similarly, the algebra posttest consisted of eight items that used pictures to resemble the manipulatives used during instruction with the corresponding number sentences written below each and eight problems that had only the number sentences like 2x + 3 = 7. Each posttest had two word problems, which required students to translate words to pictures and number sentence expressions (See Appendix G). These two word problems required students to explain, illustrate and justify their answers. They were used to analyze the level of students' conceptual understanding of fraction and algebra concepts. A rubric was used to assess the depth of conceptual knowledge.

User Survey. All of the participants completed the manipulative User Survey at the end of each unit. The purpose of this survey was to determine what students liked or disliked about each manipulative treatment. The user survey created by the researcher included eight likert type scale items and three open ended items where students were asked to write down the plus, minus and interesting aspects of using the physical and virtual manipulatives. The responses included (1) Not at all, (2) Some, or (3) A lot. Some of the questions also asked students to elaborate on their ratings, for example:

1. Do you like working with these learning tools in math? Please explain.
2. Do these manipulatives help you understand math better? Please explain.
3. Have you ever used manipulatives before? What was your experience like before? (See Appendix H).

Preference Survey. This survey was designed find out what form of manipulatives students preferred more after having used both. There were 14 items on the Preference Survey. Students had a choice of virtual manipulatives, or physical manipulatives (See Appendix I). Some examples of the statements where students were asked to choose between virtual and physical manipulatives were:

1. I can stay on task easier by using this tool.
2. I would feel comfortable working with this learning tool.

3. I can explain how to do the math better with this tool.

4. This tool helped me understand work with fraction/ algebra number sentences.

Qualitative Measures

Field notes, classroom videotapes and student interviews. During student activities in the computer lab with the virtual manipulatives and in the classroom with the physical manipulatives, the researcher interacted with students while the students were working with the physical and virtual manipulatives. Two interviewers, the researcher and a classroom teacher, asked two to four questions of several students during these interactions and also video taped each class session for the researcher to review. Students were asked a variety of open-ended questions. When students worked on fraction virtual manipulatives, they were asked questions such as: (1) Can you tell me how the number sentence is related to the virtual fraction pieces? and (2) How does the virtual manipulative help you solve the problem? When students worked on addition of fractions with physical manipulatives, they were asked questions such as: (1) Can you model and explain to me how you would add these fractions using the manipulatives? and (2) Can you explain to me what you do when you are adding and you have different denominators? These conversations were transcribed so that a written record of students' direct quotes could be used to give a better insight into students' thinking.

In addition to in-class interviews and discussions, a total of six individual interviews were conducted with three students from each treatment group, to obtain anecdotal evidence on students' experiences (See Appendix J). The researcher also took observational field notes during and after class to capture the events that occurred during the class sessions.

Data Analysis Procedures

Each research question was answered using several data sources. Table 3 provides an overview of the data sources, data analysis and the purpose for each.

Table 3.

Data Analysis Overview

Research Questions	Data Sources	Data Analysis
Question One: Achievement		
Assess Prior Knowledge	Pretest	Descriptive analysis: Means
Achievement of Physical versus virtual manipulatives	Posttest	Paired Samples *t* test
Achievement of each treatment by manipulatives and content (i.e. Physical manipulatives fraction versus virtual manipulatives fraction)	Posttest	*ANOVA* Bonferroni Post Hoc
Question One (a) Representation modes Achievement by test items: Pictorial, symbolic, word problems	Postttest	*ANOVA* Bonferroni Post Hoc
Question One (b) Unique Features of Manipulative Types	Field notes, Classroom Videotapes, Student Interviews	Qualitative analysis
Question Two: Representation Preferences for Manipulatives	User Survey, Preference Survey, Field notes, Classroom Videotapes, and Student Interviews	Descriptive statistics Frequency, percentages & means Qualitative analysis

Analyzing Quantitative Measures

Pretests and Posttests. The pretest measure was used to assess prior knowledge of the concept of fraction addition of unlike denominators and solving linear equations in algebra. First, a paired samples *t* test was performed using the scores on the posttests from the virtual manipulative treatments and the physical manipulative treatments to find if there was an overall difference in the achievement between the two treatment environments: virtual manipulatives versus physical manipulatives. In order to look at the differences in test scores among the different treatment groups and the mathematics concepts, the an Analysis of Variance, *ANOVA*, was performed on the fraction posttests from the virtual and the physical treatment groups and the algebra posttests from the virtual and the physical treatment groups. Another *ANOVA* was performed using the scores on the fraction posttests in the following subgroups: (1) pictorial test items, (2) symbolic test items, and (3) word problems. By using the repeated measures design with a cross over treatment for the two student groups, these tests allowed the researcher to compare the impact of the two modes of treatment, virtual and physical manipulatives, for each of the mathematical content area, fractions and algebra for the two groups of students.

User Survey. Numerical responses from the eight-item likert scale were evaluated by analyzing the frequencies of responses and by calculating mean rating scores.

Preference Survey. The Preference Survey responses were tabulated to determine which manipulative environment students preferred. The percentages were calculated to compare the two environments.

Analyzing Qualitative Measures

Field notes, classroom videotapes and student interviews. This study used a mixed methods approach to data analysis to thoroughly examine the impact of the virtual and physical manipulatives on students' learning of the two mathematics concepts. Understanding students' learning processes and preferences for the manipulative type was important to this study. Reviewing both quantitative findings from the surveys and qualitative findings from the interviews and observational field notes provided a more complete picture of the learning that took place in the classrooms.

In order to analyze the qualitative data, six interviews with the students were videotaped and transcribed. The researcher categorized the transcripts and notes to compare the data, formulate hypotheses, and establish patterns and/or relationships (Maxwell, 1996). In addition, the researcher kept a log of observation field notes that recorded significant events and revealing findings from class sessions. Students' comments from classroom discussions were videotaped and compiled to learn how students processed information using the two manipulative modes. Written comments from the User Survey were also analyzed and categorized by the researcher to find emerging themes in students' responses. All of these qualitative sources allowed the researcher to triangulate the data and strengthen the study by reducing potential threats to validity.

CHAPTER FOUR: RESULTS

This study used mixed method of research; therefore, the information presented in this chapter includes both quantitative and qualitative results. The quantitative data include the results of the statistical and descriptive analyses of the pre and posttests. Qualitative data presented in this chapter are results of the surveys and the interviews conducted during the study. Tables and figures are included to provide descriptive and statistical information on both quantitative and qualitative data. This chapter begins with the first research question and presents the findings from the quantitative analyses performed on the fraction and algebra pre and posttests. Next, the chapter presents findings from the analysis of subsections of the posttest to specifically examine students' work on different question types on the tests to address the question of translation between different modes of representation. Then, qualitative analysis is used to identify unique features of the two manipulative environments that may have impacted learning. The final section of the chapter addresses the second research question regarding students' preferences for the learning environments by presenting results from the surveys and student interviews.

Research Question One

The first research question was: What impact do the virtual and physical manipulatives have on students' achievement when learning concepts in fractions and algebra? To answer this question, this study used a two-phase, cross over design. In the first phase, Group One received fraction instruction using the physical manipulatives while Group Two received fraction instruction using the virtual manipulatives. In the second phase, each group received the opposite condition. That is, Group One received algebra instruction with the virtual manipulative balance scale and Group Two received algebra instruction with the physical manipulative, Hands-On Equations®.

As part of the main research question regarding achievement, the researcher also examined how the use of manipulatives facilitated the connection or translation between pictorial and symbolic notations to determine if there were any unique features that existed within the two types of manipulative environments that impacted student achievement. The subquestions of Question One were:

1. Does the use of manipulatives facilitate the connection between pictorial and symbolic notations or in terms of conceptual and procedural knowledge?

2. What unique features exist within the two manipulative environments that impact student achievement?

Several analyses were performed to answer these questions. The analysis of achievement examined (a) physical versus virtual manipulative treatments, (b) manipulative treatments and mathematics concepts (physical fraction condition versus virtual fraction condition and physical algebra condition versus virtual algebra condition), and (c) test items categorized by three different representational modes (pictorial, symbolic and word problem modes). These analyses are described in the following pages.

Analysis of Prior Knowledge

To begin the study, students took a comprehensive pretest that assessed their mathematics content knowledge of fraction addition with unlike denominators and balancing simple linear equations in algebra. The pretest included four symbolic and four pictorial items for fractions, and four symbolic and four pictorial items for algebra. The pretest also had two word problems, one on addition of fractions with unlike denominator and the other on solving an algebraic problem.

The results showed that students from both conditions had very little prior knowledge on either topic, fractions or algebra. Students in Group One scored a mean of 12.5% on the fraction section of the pretest and Group Two scored 13%. On the algebra section of the pretest, students in Group One scored a mean of 30% and Group Two scored 22%. There were no significant differences in the two student groups in terms of achievement at the beginning of the study.

Analysis by Treatment Type

Initially, the researcher was interested in determining if there was an overall difference in the achievement scores between the virtual manipulative treatments compared to the physical manipulative treatments. For the first analysis, posttest scores were categorized by treatment type regardless of mathematics concepts. That is, the posttest scores for the virtual fraction and the virtual algebra treatment were combined to determine the overall virtual manipulative posttest scores and the posttest scores when the groups used the physical manipulative fraction circles and Hands-On Equations® were combined to determine the physical manipulative posttest scores. The achievement scores were entered into SPSS and analyzed using the paired samples *t* test since each subject received both treatments and could serve as his or her own comparison. The mean scores for the posttests from the two treatment types and the results of the paired samples *t* test are presented in Table 4.

Table 4.

Mean Scores Posttests by Manipulative Treatment Types (N=36)

Manipulative Treatment Types	*M*	*SD*	*t*	*df*	*p*
Virtual Manipulatives	78.82	17.88	3.87	35	.00**
Physical Manipulatives	61.76	25.28			

p <.05. **p<.01.

Students who received instruction using the virtual manipulatives obtained a mean score on the mathematics posttests of 78.82 (*SD*=17.88) and students who received instruction using the physical manipulatives obtained a mean score on the mathematics posttests of 61.76 (*SD*=25.28). Results from the paired samples *t* test were significant, *t*(35) = 3.87, *p* = .00. Since the probability was less than the .01 level, the researcher was able to reject the null hypothesis and conclude that there was a significant difference in the achievement scores between the virtual and physical manipulative treatment types.

Analysis by Manipulative Type and Mathematics Content

Since there was a significant result from the statistical analysis of achievement by manipulative types, further analysis was performed to compare achievement scores based on manipulative treatment types and mathematics content groups. First, the results of all posttests were entered into SPSS and the descriptive information for all measures across treatment groups and mathematics contents were calculated (See Table 5).

Table 5.

Mean Performance on the Posttest by Treatment Type and Mathematics Content

Mathematics Content	Virtual Manipulative Treatment	Physical Manipulative Treatment
Algebra	83.33 (*SD* = 14.34) Group 1	80.00 (*SD* = 20.16) Group 2
Fraction	75.55(*SD* = 19.91) Group 2	45.55 (*SD* = 17.05) Group 1

The mean score on the algebra posttest for Group One, the virtual manipulative treatment, was 83.33 (SD = 14.34). Students in Group Two who used the physical manipulatives, Hands-On Equations®, scored a mean of 80.00 (SD = 20.16).

On the fraction content, the mean posttest score for Group One that used the physical manipulatives, fraction circles with the equivalence mat, was 45.55 (SD = 17.05). The mean score from the fraction assessment for Group Two that used the virtual manipulative fraction applet was 75.55 (SD = 19.91).

To further analyze the data, a 2 x 2 factorial design Analysis of Variance (ANOVA) was performed (See Table 6). This analysis was chosen because there were two independent variables or factors, manipulative types and mathematics content, each of which had two levels, virtual versus physical and fraction versus algebra and thus four groups or combinations.

Table 6.

Analysis of Variance by Treatment Type and Mathematics Concept

Source	*df*	*F*	*p*
Manipulative Types	1	15.03	.000 ***
Mathematics Concepts	1	24.11	.000***
Manipulatives x Concept	1	9.62	.003 **
Error	66		

p <.05. **p<.01. p<.001***

Results from the ANOVA produced a significant main effect for manipulative types, F(3,68) = 15.03, p < .001. This result confirms the previous paired t test on manipulative types that a statistically significant difference exists between the virtual and physical manipulative treatments on students' overall performance on the mathematics posttests. Therefore, students' scores depended on the manipulative type they used. In this particular case, the students who used the virtual manipulative treatment while learning fractions outperformed their peers using the physical fraction manipulatives.

Results from the ANOVA also produced a significant main effect for mathematics concept, F(3,68) = 24.11, p < .001. This result reveals that a statistically significant difference exists between fraction and algebra concepts on students' performance on the mathematics posttests. That is, students performed significantly better on the algebra posttests than the fraction posttests.

In addition, there was a significant interaction effect, which indicated that the effect of the manipulative treatment on the dependent variable was different depending on the mathematics concepts, $F(3,68) = 9.62$, $p < .01$. Figure 7 shows the interaction of the mean performance on the posttests across conditions.

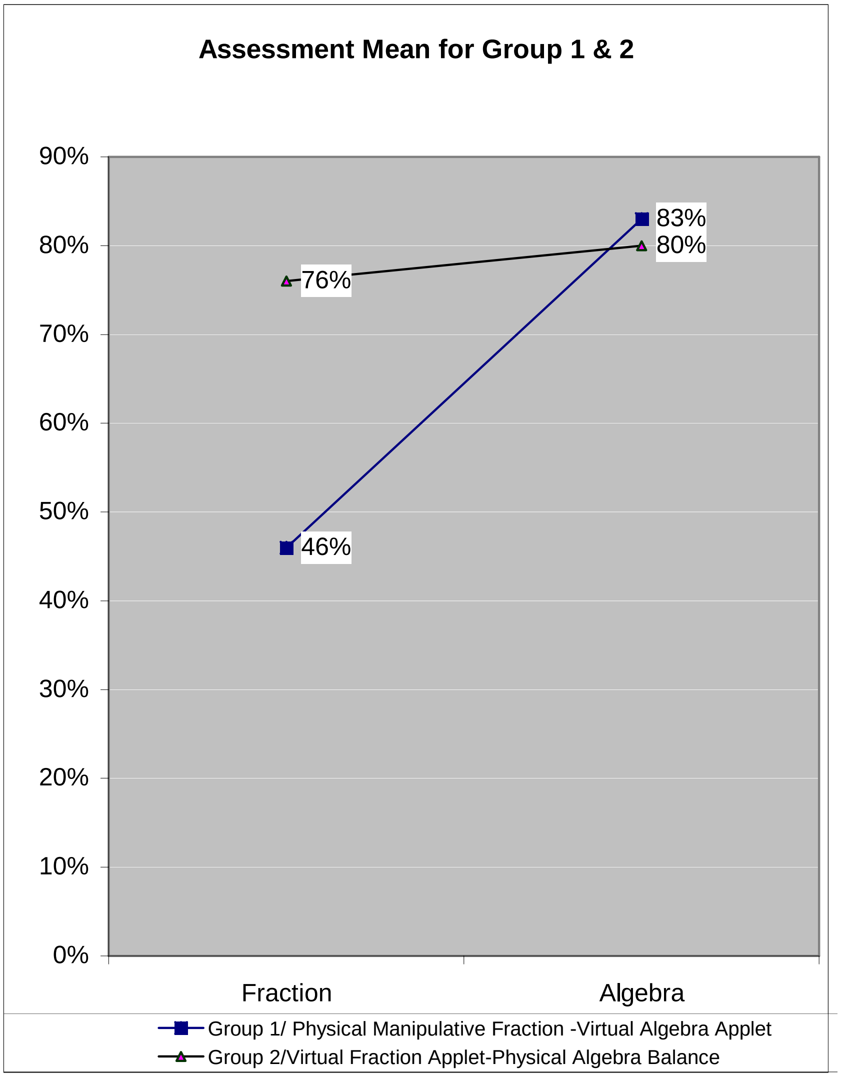


Figure 7. Line plot of the mean scores from the fraction and algebra posttests.

The ANOVA results show that the manipulative type and concepts significantly influenced the posttest scores. In addition, the interaction indicated that one variable's effect was moderated by the influence of the other. Even with a statistically significant F test from the ANOVA, one cannot tell which of the means contributed to the effect (i.e., which groups are particularly different from each other), unless more statistical analysis is performed. In order to clarify the nature of this finding, the researcher performed a post hoc test called the Bonferroni multiple comparison procedure to pinpoint where the differences existed. This allowed multiple comparisons among the different conditions.

Results of the Bonferroni multiple comparison procedure are presented in Table 7. The posttest scores from the virtual algebra and the physical Hands-On Equations® treatment groups produced non -significant results. This indicated that there were no significant differences in achievement when using virtual compared to physical manipulatives when learning how to solve basic linear equations in algebra. However, students' performance on the fraction posttest between the virtual fraction applet group and the physical fraction circles were compared and found to be statistically significant at the .001 level. This indicated that there was a significant difference in the fraction achievement scores between the two treatment groups.

Table 7.

Bonferroni Multiple Comparisons of the Posttest Means by Treatment Groups

(I) Treatment	(J)Treatment	Mean Difference (I-J)	*p*
Algebra Physical Hands-On Equations®	Algebra Virtual Balance Scale	-3.3333	1.000
	Fraction-Physical Fraction Circles	34.4444	.000***
	Fraction Virtual Applet	4.4444	1.000
Fraction-Physical Fraction Circles	Fraction Virtual Applet	-30.0000	.000***
	Algebra Physical Hands-On Equations®	-34.4444	.000***
	Algebra Virtual Balance Scale	-37.7778	.000***

*** The mean difference is significant at the .001 level.

Analysis of Test Items by Modes of Representations

The next analysis examined the differences in achievement scores among the modes of representation used for the test items (pictorial items with symbolic expression, symbolic expressions only, and word problems). There were eight pictorial items with symbolic expressions, eight symbolic only expressions, and two word problems for each mathematics posttest. These three different test item types were included so that the researcher could analyze students’ performance and make comparisons among the different representational test items. In order to do this, the posttest item scores for the fraction and algebra posttests were broken down into three categories and analyzed to investigate students' performance on each different mode of representations. The researcher compiled the achievement scores based on test items for these three categories and entered them into SPSS to perform a statistical analysis. The descriptive summary of the means for each test item group is listed in Table 8.

Table 8.

Means for Different Test Item Types

Mathematics content and Manipulative condition	Pictorial test item Means (8 problems)	Symbolic test items Means (8 problems)	Word Problems Means (2 problems)
Fractions with physical manipulatives (Group 1)	58.33 (*SD*=21.86)	22.22 (*SD*=31.37)	72.22 (*SD*=22.52)
Fractions with virtual manipulatives (Group2)	86.11 (*SD*=17.61)	70.83 (*SD*=29.70)	77.77 (*SD*=25.56)
Algebra with virtual Manipulatives (Group 1)	94.44 (*SD*=8.80)	75.00 (*SD*=26.42)	83.33 (*SD*=24.25)
Algebra with physical manipulatives (Group 2)	90.27 (*SD*=15.78).	87.50 (*SD*=20.10).	80.00 (*SD*=22.00)

By looking at Table 8, it can be shown that there were disparities among scores across test item types. For example, Group One scored the lowest on the symbolic test items for fractions while Group Two scored highest on the pictorial test items for fractions. An *ANOVA* procedure was performed on the fraction posttest scores by manipulative conditions and representational test items to see if there was statistical significance among the means. There were six sets of scores: physical manipulative treatment on pictorial items, physical manipulative treatment on symbolic only items, physical manipulative treatment on word problems items, virtual manipulative treatment on pictorial/symbolic items, virtual manipulative treatment on symbolic only items, and virtual manipulative treatment on word problems items.

The main effect for representational modes was statistically significant for the fraction posttests, $F(5,102) = 12.93$, $p < .01$. The result reveals that a statistically significant difference exists in students' performance on the mathematics posttest among the three representational test items.

A statistically significant F ratio allows for one to reject the null hypothesis that all means are equal and indicates significant differences between the groups but does not tell which groups are different from each other. Since the results from the ANOVA revealed statistically significant effects, the researcher wanted to pinpoint exactly where the differences were. To do this, a post hoc test of a multiple comparison procedure was used called the Bonferroni procedure. This procedure was performed on the fraction posttests to see differences between the types of test items. This allowed multiple comparisons among the different conditions (See Table 9). On the Bonferroni Multiple Comparisons table, only the pairs that had statistically significant probability levels are shaded. Each row corresponded to a comparison of two scores on the fraction posttest by different representational test items. For example, the first row compared the test scores of the group that worked with physical manipulatives on the pictorial items to the test scores of all other groups. There was a significant mean difference between the physical manipulative group's performance on the pictorial items compared to the symbolic only items. The mean difference was 36.11, which yielded a $p = .001$. This revealed that the physical manipulative group scored significantly better on the pictorial items than the symbolic only items. The next significant difference was between the physical manipulative treatment group's performance on the pictorial items compared to the virtual manipulatives treatment group's performance on the pictorial items. The mean difference indicates that the virtual group scored 27.77 points higher on the pictorial representational item.

The most interesting result was the second row that showed the performance of the physical manipulative group's scores on the symbolic items compared to all the other scores. There was a statistically significant difference among all the scores at the $p = .01$ level. This showed that the physical manipulative group performed significantly lower on the symbolic items compared to all other fraction test items in both groups. By looking at Table 8, it can be shown that Group One scored the lowest on the symbolic test items for fractions. The researcher examined the fraction posttest for Group One, which used the physical fraction circles and found that 11/18 students did not get any of the eight problems in the symbolic section correct. Out of the eleven, five students left the section blank and six students attempted the problems but solved all of them incorrectly and received no credit for that section. In looking at the error patterns, the researcher recognized that several students exhibited common error patterns as described in Ashlock's (2001) *Error Patterns in Computation*. Some students found the common denominators but failed to change the numerator (i.e. 2/3 + ¼ =3/12, they found the common denominator of 12 but failed to change the numerators and simply added 2+1) while others still exhibited the adding across error pattern (i.e., 1/3 + 1/5 = 2/8, where they added the numerator [1 + 1] and the denominators [3 + 5]). The poor performance on the fraction symbolic items was reflected in the low symbolic test score for Group One.

In the third row, results showed that the physical fraction group performed statistically better on the word problems compared to the symbolic items. Another result was that there was not a statistically significant difference between the physical and the virtual manipulative fraction groups on the performance of the word problems.

Detailed analysis of the multiple comparisons helped pinpoint where the statistically significant differences were among the modes of representation on the test items. For the fraction posttest, the analysis revealed that the performance on these three test item categories varied depending on the manipulative treatments.

Table 9.

Bonferroni Multiple Comparisons on Fraction Posttest by Representational Test Items

Performance on the Representational test items (I)	Performance on the Representational test items (J)	Mean Difference (I-J)	*p*
Physical Manipulative-Pictorial Items	Physical Manipulative - Symbolic Items	36.11	.001***
	Physical Manipulative - Word Items	-13.88	1.000
	Virtual Manipulative - Pictorial Items	-27.77	.033*
	Virtual Manipulative - Symbolic Items	-12.50	1.000
	Virtual Manipulative - Word Items	-19.44	.452
Physical Manipulative – Symbolic Items	Physical Manipulative -Pictorial Items	-36.11	.001***
	Physical Manipulative -Word Items	-50.00	.000***
	Virtual Manipulative -Pictorial Items	-63.88	.000***
	Virtual Manipulative - Symbolic Items	-48.61	.001***
	Virtual Manipulative - Word Items	-55.55	.000***
Physical Manipulative – Word Items	Physical Manipulative - Pictorial Items	13.88	1.000
	Physical Manipulative - Symbolic Items	50.00	.000***
	Virtual Manipulative - Pictorial Items	-13.88	1.000
	Virtual Manipulative - Symbolic Items	1.39	1.000
	Virtual Manipulative - Word Items	-5.55	1.000

* p <.05. **p<.01. p<.001***

The same analysis was performed on the algebra achievement scores, however, there were no statistically significant differences among the representational test items, $F(5,102)=1.876$, $p =.105$. The nonsignificant results signified that there were no statistical differences on the performance on the algebra posttest for each of the different test items. Therefore, no further analyses were performed.

Translation between Pictorial and Symbolic Notations

One of the subquestions for this study asked, “Does the use of manipulatives facilitate the connection between pictorial and symbolic notation?” In order to answer this question on connection between pictorial and symbolic notation, the researcher examined whether or not students were able to transfer what they had learned with manipulatives when given symbolic expressions on a test. During class instruction, students had opportunities to translate symbolic expressions to pictorial and manipulative models. Observational field notes and qualitative analysis of the achievement tests helped the researcher to formulate an answer to this question. First, the researcher looked at each test and analyzed students’ written work that revealed their solution strategies on the symbolic items, then examined students’ work and their explanations on the word problems.

Solution Strategies for Symbolic Items

The researcher examined each test and looked for evidence of students’ work to see if they relied primarily on drawings to help them solve symbolic items on the test, or if they used some form of algorithmic process to solve the problems. Analysis of the posttests showed interesting differences in the way students solved the symbolic expressions. Table 10 shows the analysis of the solution strategies from the fraction symbolic section.

Table 10.

Analysis of Students' Solution Strategies for Symbolic Items on the Fraction Posttest

Solution strategies	Group One: Physical Fraction Circles		Group Two: Virtual Fraction Applet	
	Number of Students	Percent	Number of Students	Percent
Primarily used pictorial representations	8	44.5 %	2	11%
Primarily used fraction algorithms	2	11 %	14	78%
No strategy shown	8	44.5 %	2	11%

Fraction posttest. On the fraction posttest, eight students from Group One who worked with the physical manipulatives used pictures, two used a fraction number sentence, which indicated some understanding of the algorithmic process, while eight others did not use either the pictorial or algorithmic process. In the virtual fraction group, 14 students used an algorithm that showed an understanding of the process of renaming then combining fractions, two students drew pictures and two others did not draw picture or use an algorithm. As shown in Table 10, there was a marked difference in the ways students solved the fraction problems. Group Two, who used the virtual fraction applet relied more on algorithms that were modeled on the applet by the linked representation feature. Further analysis of the posttests from the virtual fraction group revealed that most students who successfully answered the symbolic items changed the unlike fractions into fractions with common denominators, as was modeled by the virtual fraction applet. (e.g. 3/4+ 1/8= 6/8 + 1/8= 7/8). Figure 8 shows an example of how one student solved the symbolic items by using the algorithmic process.

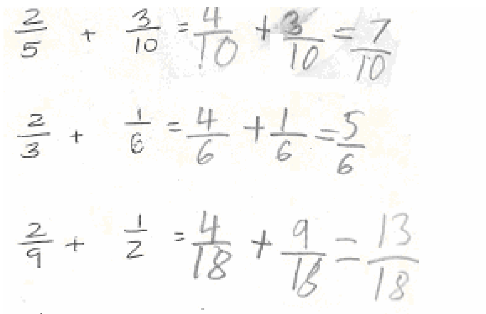


Figure 8. Student work on the fraction posttest showing an algorithmic process.

Students in Group One, who had worked with the physical fraction circle manipulatives, relied more on pictures to help them solve the symbolic items (See Figure 9).

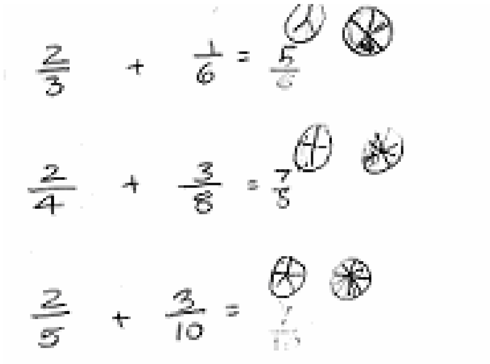


Figure 9. Student work on the fraction posttest showing a pictorial representation.

Another interesting finding was that students from Group One who used the physical manipulatives were able to solve problems that were simpler with pictures but had difficulty illustrating more complex fraction problems. To clarify this point, the researcher defined problems like ¾ + 1/8 as simpler problems because 8 is the common multiple of 4 and can be added once one of the fractions is renamed. The researcher defined complex fraction problems as problems like ¼ + 1/5, where both fractions needed to be renamed before being added. These test items also became complex for students because it was harder for them to illustrate their answers since they had to divide the fraction pieces equally into 20 fractional pieces.

Algebra posttest. On the algebra posttest, the analysis showed that most students drew pictures and showed some calculation using subtraction by crossing off the pictures when they were given only the symbolic expression (See Figure 10).

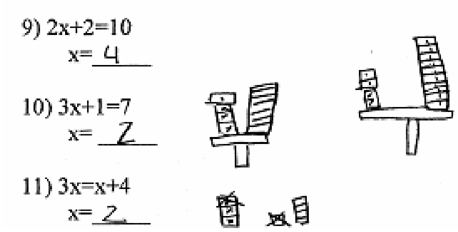


Figure 10. Student work on the algebra posttest showing a pictorial representation to solve a linear equation.

Unlike the fraction posttest, students in both groups used more similar solution strategies on the algebra posttest when presented with the symbolic test items. Table 11 shows an analysis of the solution strategies students used on the algebra posttest.

Table 11.

Analysis of Solution Strategies from the Symbolic Items on the Algebra Posttest

Solution strategies	Group One: Virtual Balance Scale		Group Two: Physical Hands-On Equations®	
	Number of Students	Percent	Number of Students	Percent
Used primarily pictorial representations	14	82 %	14	82%
Used primarily algebraic algorithms	2	12%	1	6%
No strategy shown	1	6%	2	12%

In both groups, 14 students relied primarily on pictorial representations to help them find the value of x. Only two students from the virtual manipulative group and one student from the Hands-On Equations® group used the formal algorithm of balancing equations. There was one test from the virtual group and two from the physical manipulative group that did not show either the pictorial or symbolic process. It appeared that the use of both manipulative forms gave students a way to illustrate and translate the symbolic expressions to pictorial representations for solving the problems.

Solution Strategies for Word Problems

Analysis of the fraction and algebra word problems showed that students could translate a word problem into a picture and set up the problem as a number sentence. The difference, however, existed between the fraction groups in the explanation of how they solved the problem in words (See Table 12).

Table 12.

Solution Strategies for Fraction Word Problems

Solution strategies	Group One: Physical Fraction Circles		Group Two: Virtual Fraction Applet	
	Number of Students	Percent	Number of Students	Percent
Primarily drew pictures	14	78%	0	0%
Drew picture and used algorithm	4	22%	14	78%
Used only algorithms	0	0%	4	22%

Most students in Group One, who used the fraction circles, would explain their process using the picture that they drew to illustrate the problem. One student explained, “I drew a picture and took the half and I put it in the third.” (See Figure 11).

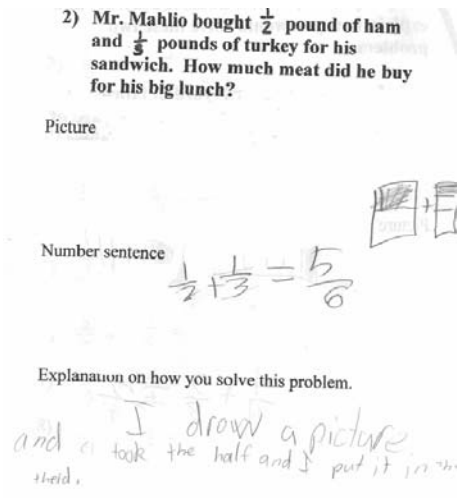


Figure 11. Example of student explanation using a number sentence and pictorial representation to solve the problem.

However, most students in Group Two drew pictures, wrote the correct number sentence and used the formal algorithmic approach to solve the problem by renaming each fraction to have common denominators. Some examples of their explanations are shown in figure 12.

- “I said to myself 2, 4, 6 and 3, 6, 9 and got my common denominator.”
- “I found a multiple of 2 and 3.”

- "I multiplied 1/3 by 2 which equals 2/6 and I divided 6 in half which is 3/6 and then I added 2/6 and 3/6 which equals 5/6."

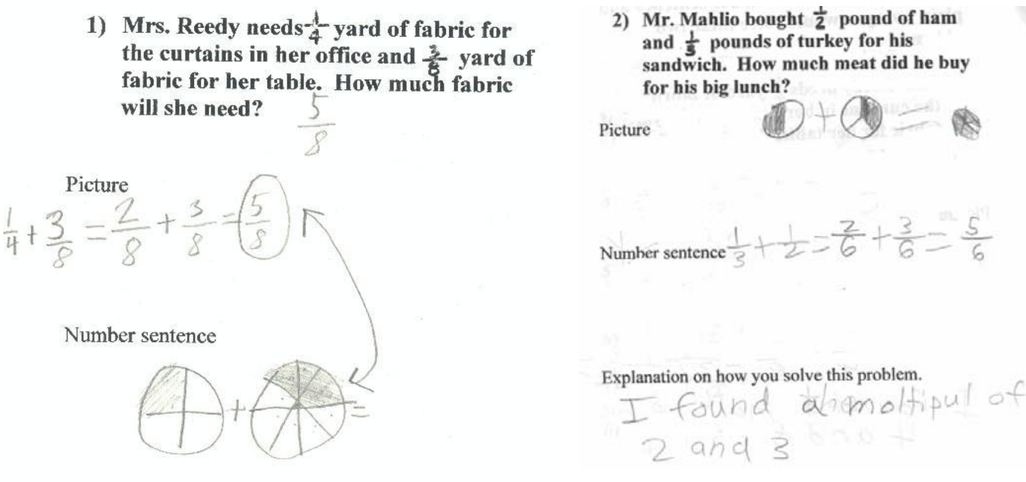


Figure 12. Examples of student solutions on a fraction word problem with pictorial and symbolic representations.

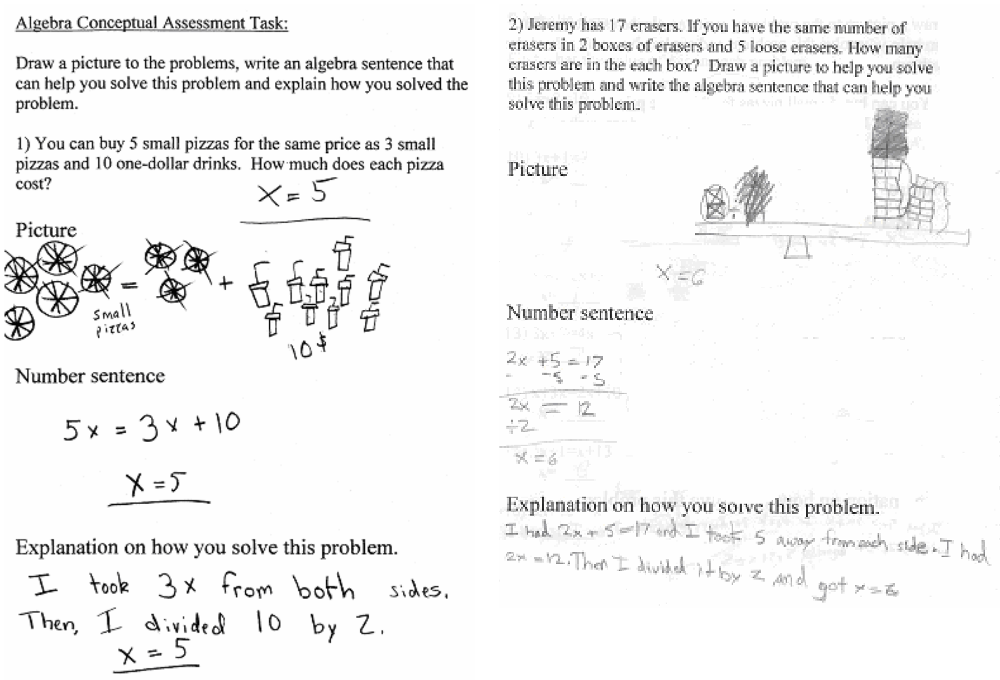
In the analysis of the Algebra word problems, the statistical analysis did not show significant differences in students' solution strategies. (See Table 13).

Table 13.

Solution Strategies for Algebra Word Problems

Solution strategies	Group One: Virtual Algebra Balance Scale		Group Two: Physical Hands-On Equations®	
	Number of Students	Percent	Number of Students	Percent
Primarily drew pictures	1	5.5%	4	22%
Drew picture and used algorithm	16	89%	12	67%
Used only algorithms	1	5.5 %	2	11%

Most of the students from both groups were able to translate the word problems into pictorial representations and a number sentence. They showed evidence of using operations such as subtraction and division to find the value of x (See Figure 13).


Algebra Conceptual Assessment Task:

Draw a picture to the problems, write an algebra sentence that can help you solve this problem and explain how you solved the problem.

1) You can buy 5 small pizzas for the same price as 3 small pizzas and 10 one-dollar drinks. How much does each pizza cost?

X = 5

Picture

small pizzas

10$

Number sentence

5x = 3x + 10

x = 5

Explanation on how you solve this problem.

I took 3x from both sides. Then, I divided 10 by 2.

x = 5

2) Jeremy has 17 erasers. If you have the same number of erasers in 2 boxes of erasers and 5 loose erasers. How many erasers are in the each box? Draw a picture to help you solve this problem and write the algebra sentence that can help you solve this problem.

Picture

x = 6

Number sentence

2x + 5 = 17
 −5 −5
2x = 12
÷2
x = 6

Explanation on how you solve this problem.

I had 2x + 5 = 17 and I took 5 away from each side. I had 2x = 12. Then I divided it by 2 and got x = 6

Figure 13. Examples of student solutions on algebra word problems with symbolic and pictorial representations.

Unique Features of the Manipulatives that Impacted Learning

This study examined the unique features that exist within the two types of manipulative environments and their learning characteristics. One particular feature that was of interest was the linking representations that existed in the fraction and algebra virtual manipulative applets in this study. Through observational field notes, student interviews and classroom videotapes, the researcher was able to find some unique features that impacted learning in each manipulative environment.

Features of the Virtual Applets

The term affordances is used in technology to describe a unique design aspect of an object that suggests how the object should be used. From the observational field notes, student interviews and classroom videotapes, there were some unique affordances in the virtual environment that impacted learning that were not as prominent in the physical environment, such as:

1. Explicit link between the visual mode and the symbolic mode;
2. Guided step by step support in algorithmic processes;
3. Unique dynamic features; and
4. Immediate feedback and self-checking system.

Explicit link between the visual and the symbolic mode. The linked representation of the visual and the symbolic mode existed in both the fraction and the algebra applets on the National Library of Virtual Manipulatives. Observational field notes from class sessions reveal that students' attention was drawn to this feature because it was one of the built-in design aspects of the applets. Here is an excerpt from the in-class interview on the day that the students used the virtual fraction applet that indicates how this unique feature helped students make a connection between the visual and the symbolic representations. The researcher asked, "Describe how the computer is helping you work through these problems." The student replied,

> When working with the first screen of the fraction addition applet, the computer asked me to rename each fraction to show common denominators. Like in this problem 1/3+ ¼ I use the arrow key to find a common denominator. I click the

> arrow key on the 1/3 fraction circle. I notice that it divides equally into sixths. So I try the ¼ fraction to the sixths. I notice it does not evenly divide into sixths so I click onto eighths where I know it's going to be even up. But I know the 1/3 fraction circle will not even up to eights. You see because you can't count by threes to get to eighths. So I keep hitting the arrow key to get to 12^{th} . I try 12^{th} on the 1/3 circle and see that the lines even up. That shows me that 12^{th} is a common denominator. After using the arrow key to divide each fraction into common denominators, I typed in the new name for the fractions. After I press the check button, I got the second screen that let me see the new number sentence next to the first sentence like this: 1/3 + ¼ = 4/12 + 3/12 =. On top I saw a picture of the fractions with the new number of pieces. I mean they are both divided into 12 parts.

In this excerpt, the student explained how she interacted between the two modes of representation to make conjectures and to confirm her answer.

This explicit link between visual and symbolic modes was also present in the virtual algebra balance scale applet. Students typed in a symbolic command such as "subtract 3x from both sides" in order to balance the equation. With that command, the dynamic feature of the applet removed 3x from both sides of the algebra scale and simultaneously displayed a new algebraic expression. Here is an excerpt from the transcripts on the day students from Group One worked with the virtual algebra balance scale.

> I have a problem 4 + 3x = 13 so I put 4 red number blocks on the left side of the balance scale with 3 blue x boxes. Then on the other side I put 13 red number blocks. When the scale balances, I know I have set up the right problem. Now I need to find out what x equals. So I first get rid of the number 4. I press the subtraction key then the number 4 then the button Go. The number of cubes disappears. Then I am left with 3x on one side and 9 blocks on the other side. So now I press the division key to divide it by 3, then press go. I am left with one x cube and 3 number cubes on the other side. And there is my answer.

In this excerpt, one can see how the student worked through each step with the virtual manipulatives and how he made sense of the numeric expression each step of the way.

Unique dynamic features. For the virtual fraction applet, one of the dynamic features was the arrow key, which allowed the user to break the fraction pieces into multiple parts. Students enjoyed clicking on the arrow key to find the common multiples. During one class session, the researcher noticed a student clicking the arrow repeatedly. When the student was asked what he was doing, he replied, “I clicked the arrow key to find that 1/3 could be renamed 33/99.” The ability to break the fraction pieces into multiple parts was a unique affordance provided by the fraction applet. This feature allowed students to experiment and test their ideas on how to rename fractions. When the researcher asked the student, “How does the arrow key help you?” She replied, “I can click on it to see that the lines even up whenever I hit a multiple of that number. For example for this fraction 1/11, I noticed that the lines evenly divided at 22, 33, 44, 55, and so on. There is a pattern in the list that I see.” The researcher followed up her response with another question, “Based on the pattern, what do you think the largest multiple you’ll find if the computer lets up go up to 100 pieces?” She answered, “ 99 would be the highest multiple because I can get there by doing 11 x 9. Here I can show you.” From the dialogue, it was clear that the student had a strong conceptual understanding of renaming fractions.

For the algebra pan balance, one of the unique dynamic features was the way the balance scale tilted and balanced based on the equation. A student from the interviews said, “I like the way the balance scale shows me I have set up the right number sentence by balancing itself. If I don’t do it right, one side slants down.” In the observational field notes, the researcher wrote, “One advantage that I saw with this tool was that the balance scale tilted as blocks were removed. This ability can help show students the inequality and equality of an equation by the tilt of the balance scale.”

Guided step by step support with formal algorithms. Both of the virtual applets chosen for this project had an emphasis on the formal algorithm of addition of unlike denominators and balancing equations. They were considered more of a concept tutorial than an exploratory tool because they had built in constraint-support systems that guided students through the algorithm. As mentioned above, the fraction addition applet is divided into a two-step process, where students must complete the step of renaming of fractions correctly before the computer will allow students to go onto the addition step. In the student interviews, one student responded, “I like how you go step by step through the fraction problem. It helps me think through the steps.” The fraction applet modeled the procedure of the formal algorithm that is used to rename fractions with unlike denominators before adding them together. Students were exposed to this procedural step each time a new problem was presented. When a student was asked to explain what she was doing when she solved the problem 1/3 + 2/9 = she replied,

> I look at the two fractions and if they don't have the same denominators, that means I can't combine them. So the first thing I need to do it to change them so that they are divided into the same number of pieces. I use the arrow key to break apart the pieces. I notice that every time I hit the multiple of 3 the lines even out. So I break it into 9 pieces. Then I type in a new name for it. 3/9. When I click the check button, it lets me go to the next step. In the new screen, I see the renamed fraction pieces and the problem on the bottom of the picture like 1/3 + 2/9 = 3/9 + 2/9 = . Now I can add them by moving the colored pieces from each fraction to the fraction sum circle or I can just add the fractions and I get 5/9.

In this explanation, the student understood the reason behind renaming a fraction before adding fractions with unlike denominators. She noticed the pattern of finding equivalent fractions, that is, that fractions can be renamed into multiples of the denominator.

The algebra balance scale applet also placed emphasis on the operations like subtraction and division to balance equations and required students to actually choose an operation and perform each step while displaying the changed equation each step of the way.

Immediate feedback and self-checking system. Both the fraction and the algebra applets had a check answer button to verify students' final answers. In addition to the final check button, the applets had several prompts in the procedure. If students entered the wrong numeric response, the computer would provide a prompt like, “The two sides don’t match the equation” or “You can’t subtract 4x from both sides unless there are at least 4xs on each side.” This immediate feedback and the self-checking button kept students from practicing the problems in an erroneous way. It also provided immediate positive reinforcement.

Features of the Physical Manipulatives

There were some unique features of the physical manipulatives that were distinct from the virtual manipulatives environment.

1. Tactile features,
2. Physical representation of the symbolic expression,
3. Opportunities for inventive strategies and mental mathematics, and
4. Over reliance on the manipulatives.

Tactile feature. One difference between the two manipulative environments was the tactile feature. Students could pick up and move the pieces easily without a mouse. With the fraction circles, students had to make the correct fraction with individual pieces and combine it with the other fraction addend. Then they used the fraction mat to find the common denominator. At times, students had difficulty with making the fractions because these fraction circle pieces were unmarked and it was hard to tell the difference between the 1/5 piece and the 1/6 piece. Sometimes the fraction pieces would fall on the floor and get lost.

The Hands-On Equations® also had a tactile feature that appealed to students who enjoyed kinesthetic learning environments. During class, students would say it was like chess because you get to take off pawns and solve problems by moving pieces off of the balance scale.

Representation of the symbolic expression to the manipulative model. Both the fraction and algebra manipulatives helped students represent the symbolic expression into a manipulative model. Although the linkage between the symbolic and the physical representations was present, it was not as closely tied together as in the virtual manipulative environment, where the symbolic expression was on the screen every step of the process. That is, for the physical manipulative group, the symbolic expression was simply written on a piece of paper. Unlike the virtual applet environment, the symbolic expression did not dynamically change with the move of the physical manipulatives. From observation, the researcher noticed that students would look at the algebraic equation, set it up on the Hands-On Equations® balance mat, then perform the procedure without any paper and pencil and without recording the process of their moves with the manipulatives as stated in this excerpt from the researcher's observational field notes:

> Students were able to see a plastic stationary pan balance that I used as I modeled the lesson. During practice time, they worked individually using a paper balance mat with the moveable pawns and the number cubes. Although I asked students to keep track of the moves they made on the balance, many of the students worked exclusively with the balance and did not record any of their moves.

Students were able to represent the fraction addition problem using fraction circles. However, the same behavior was noticed with the fraction circles and the equivalence mat. Students looked at the task sheet to set up the problem and moved the fraction circles on the mat until they found a common denominator. However, many of the students did not go through the process of renaming the fraction on paper and only wrote down the sum.

Opportunities for invented strategies and mental mathematics. Unlike the virtual fraction applet, the use of the fraction circles and mat was more exploratory in nature and allowed for more student invented strategies. For example, in the observational field notes, the researcher wrote,

> I began the lesson with simple fraction statements like ½ +1/4 and students quickly could answer that by seeing that there was a ¼ piece missing. They used the missing area to determine the sum of the addends. They seems to be looking at the residual part.

Some students were able to use this strategy to obtain the answer, but later found some problems that were too difficult to use the invented strategy. As reported in the field notes, “But when we got to a problem like 1/3 + 3/5, it was not so obvious to figure out what the remaining area represented.”

When Group Two used the Hands-On Equations®, it was interesting to note that they used a different approach to solving the problems compared to the virtual group. As mentioned before, the virtual group used the operation key to punch in subtraction to get rid of the number cubes and x cubes and used division to find the value of x. In the physical manipulative group, some students relied on multiplication to use the missing factor approach to find the value of x instead of using division. For example, if left with 3x = 6, a student might say “What times 3 gives me 6?” instead of thinking “6 divided by

3 equals 2 for x." Other students used a guess and check strategy for simple linear algebraic equations. They substituted a number for x to see if it was correct. In addition to observing invented strategies while working with the physical manipulatives, the researcher also saw more mental math being performed with simpler equations.

Over reliance on the manipulatives. One drawback the researcher noted from her observations was the over reliance on the manipulatives and the equivalence mat. Here is an excerpt from the field notes,

> Day Three**-** Physical Manipulative Fraction Unit: Over reliance on the fraction mat that helped students find the common denominator prevented some students from thinking about the relationship between the equivalent fractions and the finding common denominator. Some combined the two fraction addends like 2/6 and 3/12 and randomly placed them on the fraction mat until they saw the fraction lining up to a common denominator without much thought about how each addend must change in order to be combined using a common denominator. It seemed more like a trial and error approach where they combine the fractions and match it up. Although, class discussion time was devoted to showing students how the fraction addends could be renamed before being combined, looking at students' class work, it was apparent that they did not look at each addend individually but combined the two and placed the sum on the fraction mat.

Unlike the fraction concept tutorial, students did not get as much exposure to the formal algorithm of renaming fractions and combining the two addends with a common denominator during individual class work time. Another field note indicated,"They had difficulty with 2/3 + 1/4 using the fraction mat because they were trying every fraction CD on the mat to find a common denominator."

From these notes, the researcher observed that students over-relied on the fraction mat instead of looking at the relationships between renaming and combining fractions with common denominators.

Physical and Virtual Manipulative Limitations. There were some limitations in both representational modes. The fraction circles had limited pieces. The set that was used in class was called the Deluxe Fraction Circle Set that consisted of nine circles divided into halves, thirds, fourths, fifths, sixths, eighths, tenths, twelfths and one whole. The fraction equivalence mat had seven circles, fourths, sixths, eighths, tenths, twelfths, sixteenth and twenty-fourths. Thus, when students were given 1/5+2/3, they could not find the 1/15th fraction circle to help them. In addition, the fraction pieces could not be broken into smaller parts.

One of the limitations of the Hands-On Equations® was that it did not tilt or change with moves made on the balance; it was stationary. The researcher reported this as one of the drawbacks in this excerpt from the field notes,

> Day Four- Algebra Hands-On Equations®: One thing that I noticed as being a major drawback as I observed them work, was that the plastic pan balance and the balance mat both did not have the ability to tilt. The stationary aspect of the pan balance does not really help students correct themselves with the equation when the equation is not balanced. For example, I noticed one girl taking away a blue pawn from one side but forgot to do the same thing to the other side. Since the

balance did not tilt to show inequality she continued on with the equation and arrived at the wrong answer. If she did not check by plugging the x value back in to the equation, she would never know she got the wrong answer.

The virtual manipulatives had some drawbacks as well. One obvious problem was that there was no way to design one's own problem. There were multiple problems at different difficulty levels but they were mixed together.

Research Question Two

The second research question examined the affective nature of the study. The question read as follows, "What learning preferences exist between the virtual environment and the physical environment in teaching fractions and algebra?" In order to analyze the qualitative data, the researcher collected surveys, student interviews, and classroom observational field notes. In the following section, results from the User Survey and the Preference Survey are presented. This section also reveals some of the students' responses from the surveys that give a rationale for their choices on the survey, which help tell a more complete story of students' experiences with the different treatment conditions.

Results of the User Surveys

After each unit, students were given a User Survey that assessed their feelings towards each manipulative treatment. Students were able to select from three choices: (1) Not at all, (2) Some, and (3) A lot. Their responses were tallied and tabulated for frequency, percentages and means to compare results. Individual results from the four treatments are displayed in the following tables (See Tables 14-17).

Table 14.

Group One's User Survey Results for Physical Manipulative- Fraction Circles

(*n* = 18)

Questions	Frequency	Percentages	Mean
1) Do you like working with these learning tools in			1.88
math?	5	28%	
1=Not at all	10	55%	
2= Some	3	17%	
3=A lot			
2) Do these manipulatives help you understand math			2.06
better?	4	22%	
1=Not at all	9	50%	
2= Some	5	28%	
3=A lot			
3) Have you ever used manipulatives before?			2.56
1=Not at all	2	11%	
2= Some	4	22%	
3=A lot	12	67%	
4) I would like to use this tool again to learn other			2.11
math concepts.			
1=Not at all	1	6%	
2= Some	14	78%	
3=A lot	3	17%	
5) I can stay on task easier by using this tool.			1.89
1=Not at all	5	28%	
2= Some	10	55%	
3=A lot	3	17%	
6) Using this tool helps me correct my own			1.83
mistakes.		28%	
1=Not at all	5	61%	
2= Some	11	11%	
3=A lot	2		
7) This tool is easy to use.			1.72
1=Not at all	9	50%	
2= Some	5	28%	
3=A lot	4	22%	
8) Using this tool becomes boring.			2.00
1=Not at all	4	22%	
2= Some	10	50%	
3=A lot	4	22%	

Table 15.

Group Two's User Survey Results for Virtual Manipulative- Fraction Applets

(*n* = 18)

Questions	Frequency	Percentages	Mean
1) Do you like working with these learning tools in			2.77
math?	0	0%	
1=Not at all	4	22%	
2= Some	14	78%	
3=A lot			
2) Do these manipulatives help you understand math			2.39
better?			
1=Not at all	2	11%	
2= Some	7	39%	
3=A lot	9	50%	
3) Have you ever used manipulatives before?			1.34
1=Not at all	13	72%	
2= Some	4	22%	
3=A lot	1	6%	
4) I would like to use this tool again to learn other			2.72
math concepts.			
1=Not at all	0	0%	
2= Some	6	33%	
3=A lot	12	67%	
5) I can stay on task easier by using this tool.			2.39
1=Not at all	0	0%	
2= Some	11	61%	
3=A lot	7	39%	
6) Using this tool helps me correct my own			2.83
mistakes.			
1=Not at all	0	0%	
2= Some	3	17%	
3=A lot	15	83%	
7) This tool is easy to use.			2.83
1=Not at all	0	0%	
2= Some	3	17%	
3=A lot	15	83%	
8) Using this tool becomes boring.			1.17
1=Not at all	15	83%	
2= Some	3	17%	
3=A lot	0	0%	

Table 16

Group One's User Survey Results for Virtual Algebra Balance Scale

(*n* = 18)

Questions	Frequency	Percentages	Mean
1) Do you like working with these learning tools in math?			2.83
1=Not at all	0	0%	
2= Some	3	17%	
3=A lot	15	83%	
2) Do these manipulatives help you understand math better?			2.55
1=Not at all	2	11%	
2= Some	4	22%	
3=A lot	12	67%	
3) Have you ever used manipulatives before?			1.17
1=Not at all	16	88%	
2= Some	1	6%	
3=A lot	1	6%	
4) I would like to use this tool again to learn other math concepts.			2.78
1=Not at all	0	0%	
2= Some	4	22%	
3=A lot	14	78%	
5) I can stay on task easier by using this tool.			2.50
1=Not at all	1	6%	
2= Some	7	39%	
3=A lot	10	55%	
6) Using this tool helps me correct my own mistakes.			2.78
1=Not at all	0	0%	
2= Some	4	22%	
3=A lot	14	78%	
7) This tool is easy to use.			2.83
1=Not at all	0	0%	
2= Some	3	17%	
3=A lot	15	83%	
8) Using this tool becomes boring.			1.22
1=Not at all	14	78%	
2= Some	4	22%	
3=A lot	0	0%	

Table 17.

Group Two's User Survey for Physical Manipulative, Hands-On Equations®

(*n* = 18)

Questions	Frequency	Percentages	Mean
1) Do you like working with these learning tools in			2.61
math?	0	0%	
1=Not at all	7	39%	
2= Some	11	61%	
3=A lot			
2) Do these manipulatives help you understand math			2.44
better?	1	0%	
1=Not at all	8	45%	
2= Some	9	55%	
3=A lot			
3) Have you ever used manipulatives before?			1.05
1=Not at all	17	94%	
2= Some	1	6%	
3=A lot	0	0%	
4) I would like to use this tool again to learn other math concepts.			2.83
1=Not at all	0	0%	
2= Some	4	22%	
3=A lot	14	78%	
5) I can stay on task easier by using this tool.			2.44
1=Not at all	0	0%	
2= Some	10	55%	
3=A lot	8	45%	
6) Using this tool helps me correct my own mistakes.			2.50
1=Not at all	0	0%	
2= Some	9	50%	
3=A lot	9	50%	
7) This tool is easy to use.			2.50
1=Not at all	1	6%	
2= Some	7	39%	
3=A lot	10	55%	
8) Using this tool becomes boring.			1.27
1=Not at all	13	72%	
2= Some	5	28%	
3=A lot	0	0%	

In order to make sense of all this data, the researcher compiled all the means from the User Surveys to compare the mean rating for the different questions and statements (See Table 18). The table below is followed by a detailed analysis of the mean ratings.

Table 18.

User Survey Means Comparing Physical and Virtual Manipulatives

Questions	PM-Fraction	VM-Fraction	PM-Algebra	VM-Algebra
1) Do you like working with these learning tools in math? 1=Not at all 2= Some 3=A lot	1.88	2.77	2.61	2.83
2) Do these manipulatives help you understand math better? 1=Not at all 2= Some 3=A lot	2.06	2.39	2.44	2.55
3) Have you ever used manipulatives before? 1=Not at all 2= Some 3=A lot	2.56	1.34	1.05	1.17
4) I would like to use this tool again to learn other math concepts. 1=Not at all 2= Some 3=A lot	2.11	2.72	2.83	2.78
5) I can stay on task easier by using this tool. 1=Not at all 2= Some 3=A lot	1.89	2.39	2.44	2.50
6) Using this tool helps me correct my own mistakes. 1=Not at all 2= Some 3=A lot	1.83	2.83	2.50	2.78

7) This tool is easy to use. 1=Not at all 2= Some 3=A lot	1.72	2.83	2.50	2.83
8) Using this tool becomes boring. 1=Not at all 2= Some 3=A lot	2.00	1.17	1.27	1.22

In order to organize the results, the cell with the highest mean ratings for each statement was shaded in gray. Based on the results, students who used the virtual algebra applet rated their experience with the learning tool the highest for four out of eight questions/statements: 1) Do you like working with these learning tools in math; 2) Do these manipulatives help you understand math better; 3) I can stay on task easier by using this tool; and 4) This tool is easy to use. The virtual fraction applet and the physical Hands-On Equations® also had high ratings, which indicated satisfaction for the tools. However, the physical fraction circle, had the lowest mean rating on all the statement except for question three which actually asked students if they had used the tool before and question eight which was a negative statement that asked if the tool became boring after repeated use. These results seem to correlate to students' first response on question one, "Do you like working with these learning tools in math?" which assessed students' general satisfaction for the manipulative environments. The mean satisfaction ratings were the highest for both virtual environments: virtual algebra with a 2.83 and the virtual fraction with a 2.77. The physical Hands-On Equations® had a mean rating of 2.61. The least popular tool was the physical fraction manipulative environment with a 1.88.

An analysis of the free responses from the survey helped further explain these results. The free responses were compiled from the User Survey from the explanations students gave to questions one and two and the section where they were asked to write plus, minus and interesting features of each manipulative tool. These responses are summarized and presented below.

Virtual Fraction Applet. For the virtual fraction group, students stated that they enjoyed using the tool because it helped them learn and understand the fraction problems better. They attributed their enjoyment to the excitement of using the computer to learn mathematics and the way it made difficult concepts easier for them. Here are a few responses that describe students' experiences with the fraction applet.

"I like working on the computer with the tools because it helps me learn more."

"I can understand the problem better."

"It shows you pictures."

"It's a fun way to learn."

"At first it was a little hard but now it feels like 1st grade stuff."

"I liked it because it helps a lot with fourth grade stuff and it is also useful in challenging yourself."

Many of the students in the virtual fraction group explained that this tool helped them learn because of the visual aspects of the fraction representation and the step-by-step procedure that helped them.

" It helps me because I can see when they (the two fractions) are equal."

"I get fractions a lot more now."

“You can do it step by step.”

“I can visualize the problem.”

Students summarized that the most interesting things about using the fraction applet was: (a) using the computer in math; (b) getting help on the computer with the pictures; and (c) the feature of the arrow button, which some of them called the “magic button” that allowed the fraction pieces to break into multiple parts and the ability to move the fraction addends to the sum circle or square to check their answer.

Fraction Circles. Students from the physical fraction group enjoyed working with fraction circles and the fraction mat stating that it helped them find the answer: “I liked combining the different fraction pieces and finding the common denominator using the mat,” and “It’s fun to use these tools in math.” However, many expressed difficulty using them, “It was hard to find the right fraction pieces because they were not labeled” and “Some pieces got lost and fell on the floor.” Students reported that even though the fraction mat helped them find equivalent fractions, sometimes it was hard with “tricky” fractions like 1/3 + 2/5 because there was not a common denominator on the fraction mat for thirds and fifths.

Hands-On Equations®. Students in the Hands-On Equations® group described benefits of the tools as the following:

1. Ability to put numbers and the unknown x on and off to balance the equation (ex. “It helps me because I can take things on and off the Algebra Balance.”);

2. Visual aspect of the manipulative. (ex. “It is easier when I can see the problem instead of in my head” and “Seeing what you are working on can help a lot.”);

3. Scaffolding features. (ex. “It does it step by step to make it easier”.)

Virtual Algebra Balance Scale. Students enjoyed using the virtual manipulatives for several reasons. Some responses about the virtual algebra applet were:

“It showed the problems with red number blocks and *x* boxes.”

“The balance scale tilted and moved.”

“You can use the command keys to get rid of some number blocks and *x* boxes.”

“It helped me learn because the balance would tilt and tell me when I need to get rid of more number cubes and *x* boxes to make the number sentence balance.”

Overall students found the virtual balance scale beneficial because it helped them make the problem simpler as stated in these comments, “I like the manipulative because it helped me figure out the problem easier” and “It shows you a picture of the problem.” The most interesting aspect of working with the algebra balance scale was that they learned algebraic equations. Some of their excitement with the novelty of the tool and the concept as shown in the following responses, “It was interesting learning algebra! I thought I had to wait until I was in high school” and “ I learned that *x* can be any number!”

Results from the Preference Survey

This survey was created by the researcher to see what form of manipulatives students preferred more after having used both. Students read each statement carefully then chose either the virtual or physical environment based on which was a more true statement. There were 14 statements on the Preference Survey. From the 14, there were 12 positive statements and two negative statements. The researcher included these two negative statements to prevent students from blindly circling a preference for one on the entire list without reading each statement.

Results from the Preference Survey showed interesting differences between the two groups. Table 19 presents the results from the Preference Survey for both groups. The cells that are shaded indicate the manipulative environment that had the majority of the preference votes. Further explanation of the results follows the table.

Table 19.

Results from Student Preference Survey

Statements:	Group 1 Virtual algebra	Group 1 Physical/ fraction	Group 2 Virtual fraction	Group 2 Physical/ algebra
1. In the future, I would like to use this tool more. (+ Statement)	84%	16%	53%	47%
2. Learning with this tool is a good way to spend math time. (+ Statement)	68%	32%	61%	39%
3. It is fun to figure out how this learning tool works. (+ Statement)	63%	37%	43%	57%
4 Using this tool becomes boring.. (- Statement)	29%	69%	75%	25%
5. Working with math problems using this tool is fun like solving a puzzle. (+ Statement)	67%	33 %	50%	50 %
6. I wish I had more time to use these types of tools in math. (+ Statement)	75%	25%	47%	53%
7. Learning using this tool is interesting. (+ Statement)	65%	35%	75%	25%
8. I can stay on task easier by using this tool (+ Statement)	79%	21%	45%	55%
9. I would feel comfortable working with this learning tool. (+ Statement)	90%	10%	37%	63%
10. This learning tool make me feel uneasy and confused. (- Statement)	25%	75%	50%	50%
11. I can explain how to do math better using this tool. (+ Statement)	50%	50%	27%	73%
12. This tool was easy to use. (+ Statement)	60%	40%	50%	50%
13. This tool helped me understand work with fraction/ algebra number sentences. (+ Statement)	64%	36%	52%	48%
14. This tool help me get the right answers. (+ Statement)	69%	31%	47%	53%

Overall, students in Group One chose 13 out of 14 positive statements for the virtual algebra balance scale and one statement had a neutral response. This indicated that Group One preferred the learning environment with the virtual algebra balance scale over the physical fraction circles. Students in Group Two chose 7/14 positive statements for the physical manipulatives, Hands-On Equations® and 4/14 for virtual fraction manipulatives and 3/14 statements as neutral. Group Two showed a preference for the physical manipulative Hands-On Equations®, but the degree of preference was not as extreme as in Group One. In fact, this group had three neutral statements. An interesting finding based on the results of the Preference Survey was that both groups indicated more of a preference for the manipulative environment that they used when learning the algebra concept. That is, students' preferences for manipulatives varied depending on which group they were in when they learned the fraction and algebra concepts. Many of these statements were similar to the User Survey and showed similar results. However, there were two new themes that emerged from the analysis of the Preference Surveys, which are highlighted below.

Indication of confusion with the mathematical concept and the tool. In order to understand, why the groups had different degrees of preferences, the tenth statement in the survey helped explain the disparity. When given the statement, "This learning tool makes me feel uneasy and confused," 75 % of the students in Group One who used the physical fraction circles responded that the physical manipulative was more confusing. However, in Group Two, students responded 50% for virtual manipulatives and 50% for physical manipulatives.

In statement nine, students showed a higher comfort level with the virtual algebra applet and the physical Hands-On Equations® over the physical and virtual fraction applets. Here, the fraction circle had the lowest preference with only 10%. Although these two statements dealt with the comfort level and confusion with the tools, these responses might also indicate confusion with the concept itself. That is, results from the achievement test showed that the concept of fraction addition of unlike denominators was more challenging to learn than balancing linear equations. Based on that result, these statements might have indicated that students found the fraction concept more challenging regardless of the manipulative environment.

Help with number sentences. Students responded in Statement 13 that the virtual environments better helped them understand number sentences. In Group One, 64% indicated that the virtual algebra applet helped them understand the number sentence that appeared on the screen and 52% of the students in Group Two indicated that the virtual fraction applet helped them with the number sentences on the screen. The result from this statement further supports the idea that the virtual environments facilitated students understanding of number sentences and the symbolic algorithmic process.

CHAPTER FIVE: DISCUSSION

This study investigated the impact on students' achievement and preference when using virtual manipulatives and physical manipulatives to teach fraction and algebra concepts. The purpose of this chapter is to discuss the results of this investigation, which were guided by the following questions:

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1. What impact do virtual and physical manipulatives have on students' achievement when adding fractions with unlike denominators and balancing equations in algebra?
    a. Does the use of virtual or physical manipulatives facilitate the connection between pictorial and symbolic notation or in terms of conceptual and procedural knowledge?
    b. What unique features exist within the two types of manipulative environments that impact student achievement?
2. What representation preferences exist between the virtual environment and the physical environment in teaching fractions and algebra?

From the inception of this study, the researcher's purpose was to investigate whether there were achievement differences when using two different manipulative environments, and to delineate unique features or learning characteristics that may explain the differences.

Based on the research questions of the study and after careful analysis of the quantitative and qualitative data collected, seven major conclusions were drawn. These conclusions are presented and explained in detail in this section.

Conclusion 1: Students in the virtual manipulative group outperformed students in the physical manipulative group when learning the mathematics content.

Conclusion 2: Students in the virtual manipulatives fraction treatment group had scores on the posttest that were statistically higher than students who worked with the physical manipulative fraction circles.

Conclusion 3: Students who worked with the virtual manipulative algebra balance scale had posttest scores that were not statistically significant from students who worked on the physical manipulatives, Hands-On Equations®.

Conclusion 4: Students in the virtual manipulative fraction treatment group performed significantly better on pictorial and symbolic posttest items than students in the physical manipulative fraction group and demonstrated better understanding of the procedures for renaming and adding of unlike denominators.

Conclusion 5: On the fraction posttest, conceptual understanding, as measured by the ability to solve word problems and to translate from word to pictorial to symbolic representations, was not statistically significant between the virtual and physical manipulative groups, however solution strategies differed between the two treatment groups.

Conclusion 6: There were unique features in the virtual manipulative environment that provided guidance for learning formal algorithm for adding fractions with unlike denominators, such as, (a) linked representations, (b) step-by-step procedures, and (c) immediate feedback systems.

Conclusion 7: Students preference for a tool did not depend on whether it was a virtual manipulative or a physical manipulative, but was determined by students' learning experiences with the specific applet, manipulative tool and mathematical concept.

The following sections elaborate on these major conclusions. The first section addresses the first research question regarding the impact on student achievement by synthesizing statistical analyses presented in the previous chapter. It includes a discussion regarding the different features within each environment that may have helped or hindered students' learning. The next section discusses the outcomes of the Preference Surveys and User Surveys and how they are related to teaching and learning with manipulatives. The remaining portion of this chapter discusses the general implications of this study to classroom teaching and learning, the limitations of the study, and the implications for future research.

Impact on Student Achievement

Discussion of the Impact of Manipulative Type & Mathematics Concepts

Based on the results of the posttests, this study revealed several interesting findings. First, results from the statistical analysis showed that there was a significant difference between learning in the virtual and the physical environments. That is, the study showed that overall, the groups that worked with virtual manipulatives did significantly better than the groups that worked with physical manipulatives. Yet, deeper analysis revealed that the effectiveness of the specific applets and physical manipulatives depended more on the mathematical concepts the students were learning when they were using the specific manipulative. For example, students who used the virtual fraction applets scored significantly higher on the posttests than students who used the physical fraction manipulatives. However, students who used the virtual balance scale had scores on the posttests that were similar to the students who worked with the physical Hands-On Equations®. The significant result from the fraction unit indicated that the fraction applet provided a more effective learning environment than the physical fraction circles for learning fraction addition with unlike denominators. The non-significant result from the algebra unit was just as revealing in that it indicated that physical and virtual manipulatives could be equally effective for teaching students to solve linear equations.

One explanation for the difference in achievement in the fraction unit may be attributed to the fraction applet feature that allowed students to model the algorithmic procedures. This was evident in students' work on the posttests where students rewrote the problem 1/3 + 1/4 as 4/12 + 3/12 =. The virtual fraction applet gave students more opportunities than the physical manipulatives to concretize their understanding of the renaming procedure with the step-by step procedure. In addition to the support with the algorithmic process, students received immediate feedback that reinforced their learning of the algorithm. With a challenging concept such as addition of fractions with unlike denominators, the virtual tool provided more scaffolding that allowed students to understand the concept of renaming a fraction to add two fractions with a common denominator. Kaput (1995) explains that the problem with physical manipulatives is that people cannot keep record of everything. When students manipulated the physical fraction pieces, the task of adding fractions with unlike denominators may have presented too much cognitive overload.

In learning to solve linear equations, both manipulative environments provided similar support for student learning. Although, there was an explicit linked representation on the applet that showed the symbolic expression in the virtual algebra balance scale, posttests from both groups showed that students relied more on the pictorial model than the algebraic algorithm. That is, students used the picture of the algebraic expression on the balance scale to cross off number cubes and *x* cubes to find the value of *x* rather than use the algebra algorithm.

Discussion of Different Modes of Representation

The posttests were designed to examine students' performance in three representation modes: pictorial, symbolic and word problems. Analysis of the means for each test item type revealed some interesting differences between the treatment groups. On the fraction test items, students in the virtual manipulative group scored significantly better on the pictorial and symbolic only items than the students in the physical manipulative group. For both treatment groups, the pictorial assessment item scores were higher than the symbolic only assessment scores. This difference indicates that students had more difficulty with the problems when there were no pictorial representations of the problems.

An interesting difference was also observed in the disparity of the scores for Group One, which used the physical fraction circles, among the three test item types. There were statistically significant mean differences between pictorial and symbolic

(36.11 percentage points), and between symbolic and word (50.00 percentage points). Students performed best on the word problems, then the pictorial items, and worst on the symbolic items. This might indicate that when students were given a context and a chance to draw a pictorial representation to help them solve the fraction word problem items, students performed significantly better than when only given symbolic equations. Analysis of the symbolic test items revealed that 61% of the students either left the entire section blank or answered the section incorrectly. The students who left the entire section blank may not have felt confident in their ability to tackle the problems without the aid of the pictorial representations. In addition, students who answered incorrectly indicated some misconceptions of the procedural method.

Analysis of the posttests showed that the two groups relied on different solution strategies. That is, Group One who used the fraction circles, relied heavily on drawing pictures to solve fraction problems with unlike denominators. Group Two, who used the virtual fraction applet, relied more on formal algorithms of renaming fractions with common denominators before combining the two addends. The analysis also revealed that students in Group One were able to draw pictures to help them when the fraction problems were simpler. However, when the problems involved more complex fractions, they had difficulty using the pictorial model to solve the problems for problems like 2/9 + 1/2 or ¼ + 1/5.

Performance on the fraction word problems designed to assess conceptual understanding by analyzing students’ ability to translate from word to pictorial to symbolic modes was not statistically significant between the virtual and physical manipulative groups. Interestingly, each groups' word problem achievement scores were higher than their overall posttest scores. This may suggest that the word problems provided students with a context for the problem. Asking students to illustrate the fraction problem and write a number sentence may have assisted them in making sense of the problem. Students may have used the connection to real world, pictorial and symbolic representations to help them solve the word problems. According to Lesh, Post, and Behr (1987), students who can easily translate from one representation to another are able to use the representations as a tool to approach problems from several different perspectives. An alternative explanation for the higher performance on the word problem for the physical fraction circle group may have been due to the ease of the word problems. The two word problems translated into numeric fraction sentences were ¼ + 3/8 and ½ + 1/3 which may have been easy enough to solve by simply drawing a picture or looking at the residual part of the fraction. This may indicate that the fraction circles were equally effective as the virtual fraction applet in introducing the concept of simpler forms of fraction addition with unlike denominators.

Discussion of Unique Features

Although the physical manipulatives and the virtual manipulatives were matched to be as similar in nature as possible, the virtual applets had capabilities that the physical manipulatives did not. That is, the virtual fraction circles or squares could subdivide into multiple parts, which allowed students to see how fractions are renamed. The algebra balance virtual applet tilted when variables or numbers were added onto the balance, while the physical Hands-On Equations® manipulative pan balance remained stationary.

Although both treatments provided some form of visual and symbolic representations, the virtual applets had numeric sentences that appeared on the same screen as the manipulatives. This linking feature may have allowed students to better connect their work between the applets and the symbolic notations in the virtual environment.

Unique features of physical manipulatives. The analysis from class observational field notes and student interviews was used to find some unique features in each environment. In the physical environment, the unique features included:

1) Tactile feature,

2) Physical representation of the symbolic expression,

3) Over reliance on the manipulatives, and

4) Opportunities for inventive strategies and mental mathematics.

The tactile feature of physical manipulatives appeals to kinesthetic learners who prefer active engagement. Much research has been shared in the education community about the value and effectiveness of using physical manipulatives, and it is almost accepted belief that good mathematics teachers use these tools to build conceptual understanding (Clements & McMillen, 1996; Sowell, 1989; Thompson, 1994; Sobol, 1998). Using manipulatives allows students to translate their understanding from a physical model to a symbolic expression, which can scaffold their learning.

Similar to the results from the Rational Number Project (Cramer. Post, & DelMas, 2000), students in this project used inventive strategies, like finding the residual part to solve addition of fractions with unlike denominators. In the Rational Number Project, when students compared two fractions like 3/4 and 5/6, they looked at the “residual piece," the piece that was left over or empty, to compare fractions. Similarly in this study, students in the physical manipulative fraction circle group said, "I know ½ + ¼ = ¾ since there is a ¼ piece missing." In addition to observing inventive strategies while working with the physical manipulatives, the researcher also saw more mental math being performed with simpler operations. For example with the Hands-On Equations®, some students used a guess and check strategy for simple linear algebraic equations. They substituted a number for *x*, and used mental math to check if it was correct.

This study also found a problem with over reliance on the manipulatives. For example, the group that used the fraction circles and the equivalence mat over-relied on the tools and did not make connections to the formal algorithmic process to help them add fractions with unlike denominators. Kaput (1989) noted that sometimes the connection between the action on the manipulatives and the symbolic notation are unclear, and that work with physical manipulatives creates a cognitive overload. In this study, students who used the fraction circles to add fractions with unlike denominators, had difficulty keeping track of the procedures in their head and failed to see the connection between their manipulation with the fraction pieces and the symbolic notations.

Unique features of virtual environments. In the virtual environment, these unique features were revealed:

1) Explicit link between the visual mode and the symbolic mode.

2) Guided step by step support in algorithmic process.

3) Unique dynamic features.

4) Immediate feedback and self checking system.

This research supports some of the conclusions drawn from previous research. For example, Izydorczak (2003) reported that the linked representations had several potential benefits.

> It (linked representation) can supply feedback to the user in a graphic representation to help the user understand changes s/he made in the symbolic representation. It can force the user to a level of conscious awareness by requiring symbolic representation in order to get concrete results. It can support learners at different conceptual levels. As a result, linked representation may play an important role in making symbolic representations concrete for students. (p. 214)

From the analysis of the posttest items, there was evidence that students who worked with the virtual fraction applets used the algorithmic process. The researcher examined the test papers to see how students solved fraction addition with unlike denominators and found that many students renamed the fraction and rewrote the fraction addition problem (e.g., 1/3 + ¼= 4/12 + 3/12 = 7/12). In addition, during interviews, students were able to explain why they needed to rename the fraction, which showed that they understood the concept behind renaming before combining fractions with unlike denominators.

Renaming and combining fractions with common denominators was the approach that was emphasized in the virtual manipulative environment, but was not present in the physical manipulative environment. In the fraction posttests students in the virtual fraction applet treatment were able to transfer the learning of the algorithm to help them solve fraction problems. The results from this study relate to research done by Fennema (1972) who compared groups of students taught symbolically versus those taught using physical manipulatives. Her research found that those who learned using symbols outperformed those using manipulatives on a test for transfer.

Discussion of Algorithmic Thinking

An interesting matter of discussion is raised by the result of this study on the topic procedural understanding, or more specifically, on algorithmic thinking. Although, many researchers caution against premature introduction of formal algorithms in elementary grades (Kamii & Dominick, 1989), many of the concerns stem from students learning the procedures without sufficient conceptual development. They fear that students focus on memorizing steps instead of logically solving the problem. The concern is that students never learn the algorithm or carry out an algorithm without understanding why they perform certain steps. Basically, the argument is that exclusively teaching students steps to an algorithm is devoid of meaning (e.g., the invert and multiply rule for division of fractions).

In this study, the fraction applet promoted algorithmic thinking because students learned the procedure while building a conceptual foundation for fraction addition with unlike denominators using the dynamic visual representation. According to Mingus and Grassl (1998), algorithmic thinking is:

> A method of thinking and guiding thought processes that uses step-by-step procedures, requires inputs and produces outputs, requires decisions about the quality and appropriateness of information coming in and information going out,

> and monitors the thought process as a means of controlling and directing the thinking process. In essence, algorithmic thinking is simultaneously a method of thinking and a means for thinking about one's thinking. (p. 34)

Using this definition, the virtual fraction applet offered many of these opportunities such as guiding thought processes, using a step-by-step procedure, and having students put in numbers and get outputs. Probably the most important aspect of algorithmic thinking as described above is that students are given the opportunity to think about the method and reflect on their thoughts. It is critical to note that the researcher provided discussion time at the end of every class session for students to express their thoughts about what they learned. In this way, she promoted the meta-cognitive aspect that is necessary in promoting algorithmic thinking.

The proper use of the fraction applet provided students with the conceptual knowledge and the procedural knowledge of adding fractions with unlike denominators. The National Council of Teachers of Mathematics (2000) supports the development of algorithms and computation fluency by stating,

> Computation fluency refers to having efficient, accurate, and generalizable methods (algorithms) for computing that are based on well-understood properties and number relationships. Students should come to view algorithms as tools for solving problems rather than as the goal of mathematics study. As students develop computational algorithms, teachers should evaluate their work, help them recognize efficient algorithms, and provide sufficient and appropriate practice so that they become fluent and flexible in computing. (p. 144)

Learning Preferences between Virtual and Physical Manipulatives

On the User Survey, students' ratings for the virtual algebra balance scale, virtual fraction applet and Hands-On Equations® were higher than the ratings for the physical manipulative fraction circles. Students from both virtual environments reported that the linked number sentences helped them understand the math. They also felt that they were more on task with the virtual applets than the physical manipulatives. This was similar to the results of Drickey's (2000) study that looked at students' attitudes towards virtual manipulatives and found high levels of on-task behavior and positive comments from students about how they enjoyed this method of instruction.

Students also reported that the virtual algebra and fraction applets helped them correct their errors. Students recognized that these two environments had check buttons and these features helped them recognize mistakes.

The three manipulatives that impacted student achievement, the virtual algebra balance scale, virtual fraction applet and Hands-On Equations®,were all new to the students. This newness may have created a novelty effect. Question Three examined which tool had the most prior exposure for students. It asked, " Have you ever used manipulatives before?" The manipulative that had the most prior exposure was the physical fraction circles. Most students had not seen the National Library of Virtual Manipulatives and the Hands-On Equations® gear prior to this study. As a result, students were excited to use these new tools in their mathematics classrooms. In fact, students reported in the free response section of the survey that they enjoyed using Hands-On Equations® because it helped them learn a new topic in mathematics. They were excited to tell their friends that they were learning algebra. Other students were envious because they thought algebra was a high school course. This novelty effect could also explain why students indicated that the physical fraction circles were boring to use, since they had seen and used this manipulative before.

Based on the Preference Survey, students seemed to prefer the tool that gave them the most success with the concept they were learning. In the class that had the virtual tool for the more difficult fraction concepts, students rated that they preferred the physical manipulative, Hands-On Equations®. In the class that had the physical tool for the fraction concept, students stated that they preferred the virtual algebra balance applet over the physical manipulatives. This showed that regardless of the manipulatives types, students preferred the treatment where they felt most success. In this case, their familiarity with the mathematics concept they were learning also played an important role in determining their preference for a tool.

Limitation of the Study

A number of limitations should be noted in the present study. First, time of instruction could have been longer to allow for students to practice the concepts of addition of fractions with unlike denominators. It was a difficult concept for many third graders. Second, the preference survey also shows evidence of this novelty effect. Because students had not used the virtual algebra balance scale, virtual fraction applet and Hands-On Equations® prior to the study, the newness of these tools may have influenced students' preference for using them. Third, the population of the study included only two third grade classrooms of 18 students each. Therefore, the results are not representative and generalizeable to all third graders. Third, since the researcher was both the instructor and the interviewer, students may have been hesitant to express negative responses during student interviews. Since the instructor was busy teaching and helping students, she might have missed observing other aspects of classroom episodes that a third party may have noticed.

Implication for Classroom Instruction and Recommendations

This study supports the idea that teachers need to be selective in choosing appropriate virtual tools for teaching specific mathematics concepts. It also suggests that certain manipulatives and models are more effective in illustrating certain concepts. Physical and virtual manipulatives can work in a complimentary manner and be used for different purposes by offering different unique affordances. Some virtual manipulative applets that are designed as concept tutorials can help students learn formal algorithms while giving them the support they need with pictorial representations. Some physical manipulatives might be used for exploratory reasons when teachers want to promote more inventive thinking strategies. From this study, use of the fraction circles did not prove to be as effective in developing students' ability to perform the algorithm for addition of fractions with unlike denominators. However, it allowed students to be more inventive with their solution strategies and helped them solve simpler fraction problems with pictorial representations. The problem occurred when students were given more complex problems, which they could not illustrate easily. Based on some of these analyses, the physical manipulatives, like fraction circles can be effective in introducing the concept of addition with fractions with unlike denominator. Eventually when students are introduced to an efficient algorithm for fraction addition with unlike denominators, the virtual fraction applet can be a very effective tool that allows students to connect the pictorial representation with the symbolic notations. Teachers must consider the needs of individual students and identify the best teaching methods and tools to meet the needs of different learning styles.

Whether a teacher chooses a virtual manipulative or a physical manipulative to teach a mathematical concept, s/he must understand that actions with the tools are not an automatic guarantee for learning. Teachers must provide students with time to discuss their sensory or virtual experiences with the manipulative models in a reflective manner so that they can make mathematical connections. The important point, therefore, "is not the manipulation of objects in itself that is important to children's learning. What is important is the mental action that is encouraged when children act on objects themselves" (Williams & Kamii, 1986, p. 26).

Implications for Research

This research was unique from past studies on virtual manipulatives because it focused specifically on using a virtual concept tutorial on a mathematics topic that relied heavily on procedural understanding (i.e., the addition of fraction with unlike denominators). The unique affordances of the virtual fraction applet, like the step-by-step procedures, the interactive dynamic images, and the immediate feedback system facilitated the learning of the procedures for the algorithm. This study makes an important contribution to the research on virtual manipulatives by highlighting that certain features of individual virtual manipulatives may be very effective for enhancing student achievement for some mathematical concepts, while other virtual manipulatives may have the same impact on student achievement as physical manipulatives for other mathematical concepts.

Further study is needed to learn the effectiveness of other virtual manipulatives and physical manipulatives in teaching specific mathematics concepts. As Kaput (1992) argued, computer-based "manipulatives" may have some advantages over physical ones because of the computer's ability to record processes, or to display variables. Research should be conducted on other virtual concept tutorials to see if they can help facilitate learning of complex algorithms while providing conceptual understanding. In addition, one might consider investigating how research on virtual manipulatives can help software developers and textbook publishing companies create mathematics applets that are rich in building conceptual understanding and procedural fluency.The role of technology within school mathematics is still very ill-defined but holds many promising leads. Further research needs to be done on integrating more technology in the teaching and learning process to ensure that our students are well prepared for the future.

Implications to Instructional Designers and Curriculum Developers

There are increasing amounts of virtual manipulatives being developed by computer programmers, publishing companies and independent educational businesses. As the development of virtual manipulatives advances, instructional designers must work with educators to carefully review the benefits and drawbacks of different virtual applets for teaching specific concepts. There are and will be some virtual manipulatives that surpass the capabilities of their physical counterparts and have features that can only exist in the virtual realm. This study shows that the virtual manipulative fraction adding applet supported students learning of the algorithms with the step by step process, interactive visual images, and immediate and specific feedback. This can be used to guide instructional designers to develop other meaningful applets that may help students learn other procedures in mathematics.

In addition, this particular virtual applet had a linking feature which allowed students to interact with both the symbolic and visual representational modes simultaneously, unlike most computer manipulated programs, which allow users to interact only by typing in the correct numeric responses. Previous research indicates (Lesh, Landau, & Hamilton, 1983) that it is the ability to make translations among and between the modes of representation that makes mathematical ideas meaningful to learners, curriculum developers and instructional designers should investigate ways to integrate multiple modes of representation in a learning environment. For example, a concept tutorial should be designed to have the linked representations with dynamic images and real-life contextualized problems. This would offer learners exposure to three out of the five modes of representations that has greater potential for enhancing mathematical learning.

Conclusions

From this study, results showed that the fraction applet had features that scaffolded students' understanding of the algorithmic process of addition of fractions with unlike denominators. Features such as the step-by-step procedures, interactive visual images and the immediate feedback impacted the fraction posttest scores between the virtual and physical treatments. However, students' achievement did not differ between the virtual algebra balance scale group and the Hands-On Equations® group. The general conclusion, based on these results, is that there are certain virtual applets that can make a difference in students' understanding of a concept while other virtual manipulatives may be just as effective as their physical counterparts for impacting learning.

An important learning characteristic that emerged from this study was that the fraction applet promoted algorithmic thinking. Developing algorithmic thinking produces students who understand their methods and can carry them out proficiently so that they can think about more important things, such as why they are doing what they are doing and what their results mean. Algorithmic and procedural thinking may improve students' mental arithmetic skills, help them understand operations, and develop sound number sense.

According to the National Research Council in *Adding It Up,* developing proficiency with rational numbers requires " instructional materials that support teachers and students so that they can explain why a procedure works rather than treating it as a sequence of steps to be memorized" (p. 240). They also state that,

> The need is for students to understand the key ideas in order to have something to connect with procedural rules. For example, students need to understand why the sum of two fractions can be expressed as a single number only when the parts are of the same size. That understanding can lead them to see the need for constructing common denominators. (p. 241)

These quotes were realized in this study. First, this research set out to examine whether the virtual or physical manipulatives would be effective instructional tools in supporting students in learning fraction addition and solving algebraic expressions. The study revealed some distinct differences in student achievement and learning characteristics. The major benefit of the fraction applet, in particular, was the linked representation that provided the scaffolding necessary to allow students to focus both on why the procedure worked as well as the sequence for the procedural steps. As stated in Moyer, Bolyard & Spikell's (2002) definition of virtual manipulatives, the tool was interactive and the visual representation of a dynamic object presented opportunities for students to construct mathematical knowledge. In other words, the virtual fraction applet allowed students to engage in higher-level thinking as they worked through the problems and to construct mathematical knowledge while using the visual and symbolic representation of the procedure of adding fractions with unlike denominators.

The significance of this research is that it highlighted certain features of virtual manipulatives, which facilitated mathematics learning. It supports the statement made by the National Council of Teachers of Mathematics that, "Students can learn more mathematics more deeply with the appropriate and responsible use of technology. They can make and test conjectures. They can work at higher levels of generalizations and abstractions" (NCTM, 2000, p. 7). The appropriate use of the virtual fraction applet allowed for students to make and test conjectures about finding common denominators and to connect the fraction addition algorithm with visual representations to facilitate their understanding of an abstract concept. Future research should continue to explore the academic benefits of using virtual manipulatives in mathematics teaching and learning.

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APPENDICES

APPENDIX A: LETTER OF INFORMED CONSENT-PARENT

Letter of Informed Consent-Parent

This study is being conducted to investigate the use of instructional strategies that can be used in mathematics classrooms, specifically the use of virtual and physical manipulatives.

Your child will be asked to participate in two weeks of instruction learning about addition of fractions and primary algebra concepts. Students will benefit from learning fraction and algebra concepts using two types of instructional strategies: physical manipulatives and computer based manipulatives called virtual manipulatives. This research will add to the literature and findings on how to effectively use instructional materials to help students build procedural and conceptual understanding of mathematics. There are no foreseeable risk.

All data collected in this study will be confidential; all person-identifiable data will be coded so that your child cannot be identified. Students who participate in short interviews will be taped. The tapes will be used to transcribe notes then will be destroyed.

Your participation is voluntary and you may withdraw from the study at any time and for any reason.

This research is being conducted by Jennifer Suh, Doctoral Candidate at George Mason University under the direction of Dr. Patricia Moyer-Packenham at George Mason University. She may be reached at (703) 993-3926. You may contact the George Mason University Office of Sponsored Programs at (703) 993-2295 if you have any questions or comments regarding your rights as a participant in the research.

This research has been reviewed according to George Mason University procedures governing your participation in this research. In addition, this project has been approved by Loudoun County Public Schools and Kristen Fields-Reedy, Principal of Little River Elementary.

Consent
I have read this form and give permission for my child to participate in this study.

Parent's Signature: ____________________________
Child's name: ________________________________
Date of Signature:_____________________________

APPENDIX B: LETTER OF STUDENT ASSENT

Letter of Student Assent

Hi. This is Mrs. Suh from the third grade. I am doing a study to find better ways to help students learn math using tools called virtual and physical manipulatives.

For two weeks, you will learn about fractions and algebra using two different tools: physical manipulatives and computer based manipulatives called virtual manipulatives. This will not be part of your grade but you will help me learn how children learn math. I will collect every test and activity sheet that you work on so that I can learn more about the way kids learn but your name will not show up in any of my work. You may be taped if you are in a short interview with me. The tapes will be used to record notes then will be destroyed. If you do not want to be in this study, you can let me know at any time for any reason.

I will be working with Dr. Moyer from George Mason University. Mrs. Reedy, our principal and Loudoun County Public Schools have given me permission to do this study in your class. I look forward to working with you on this study and thank you for your help.

Student Assent:
I have read this form and I would like to be in the study.
Student's Signature: ___________________________
Child's name: ________________________________
Date of Signature:_____________________________

APPENDIX C: LETTER OF APPROVAL FOR RESEARCH FROM LOUDOUN COUNTY PUBLIC SCHOOLS

Loudoun County Public Schools
Office of Research
102 North Street, N.W.
Leesburg, Virginia 20176
Telephone: 703-771-6415
FAX: 703-771-6409

TO: **Jennifer Suh, Third Grade Teacher, Little River Elementary**

FROM: **Sharon Ackerman, Assistant Superintendent for Instruction**
Stephan Knobloch, Research Supervisor

RE: **Dissertation Proposal**

Date: **March 29, 2004**

Your request to conduct a study with third grade students at Little River Elementary School has been reviewed and approved. The approval is limited to the scope (virtual and physical manipulatives for two units of study) and the scale (two classrooms) of the proposal mentioned in your abstract. The results of your study will have value to other elementary math teachers in Loudoun County Public Schools as they explore the effectiveness of virtual manipulatives and non-linguistic representation of abstract concepts.

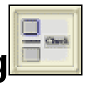
Please share your findings with Cheryl Wimer, Math Supervisor and consider sharing with your colleagues in Loudoun County Public Schools.

Good luck with your study.

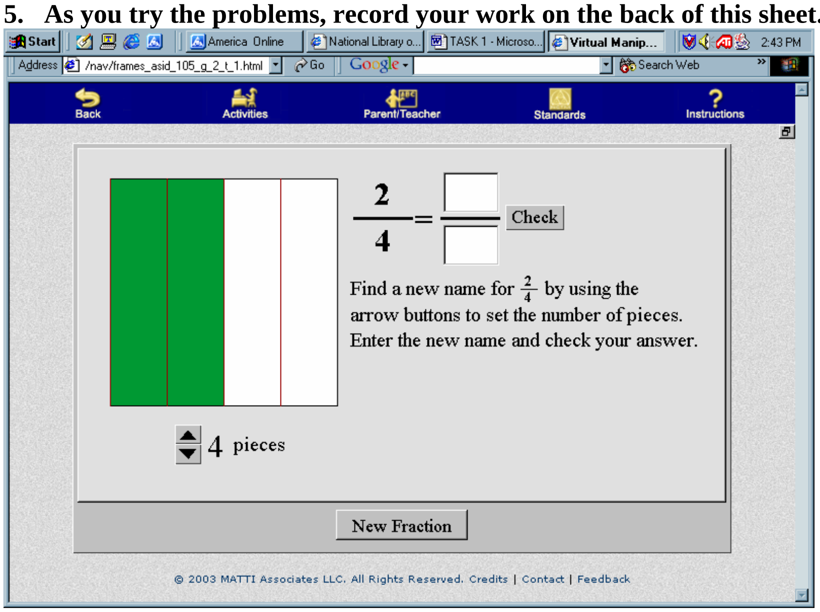
APPENDIX D: TASKSHEETS FOR FRACTION AND ALGEBRA INSTRUCTION

Fraction- Virtual Manipulative Task Sheet 1

Name: ________________Period___Date: ______

TASK 1: Renaming 

1. **Go to the NLVM at http://matti.usu.edu/nlvm/nav/vlibrary.html**
2. **Click on Number sense grade 3-5**
3. **Click on Equivalence fraction.**
4. **Try 5 problems.**
5. **As you try the problems, record your work on the back of this sheet.**



Renaming a fraction means finding an equivalent fraction.

To complete this activity:

1. **Use the arrow keys to divide the whole unit into more or fewer parts.**
2. **Stop when there are red lines on top of each of the edges of the colored parts.**

The numerator is the number of colored parts. The denominator is the total number of parts that make up the whole unit.

3. **Enter the name of the equivalent fraction into the boxes to the right of the name of the original fraction.**
4. **Check whether or not you have an equivalent fraction by clicking the "Check" button.**
5. **Find another equivalent fraction by increasing or decreasing the number of parts that make up the whole unit.**

Record your work.

Can you make a rule for finding equivalent fractions?

____

____

____

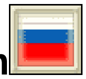
____

* What do you think about today's lesson?

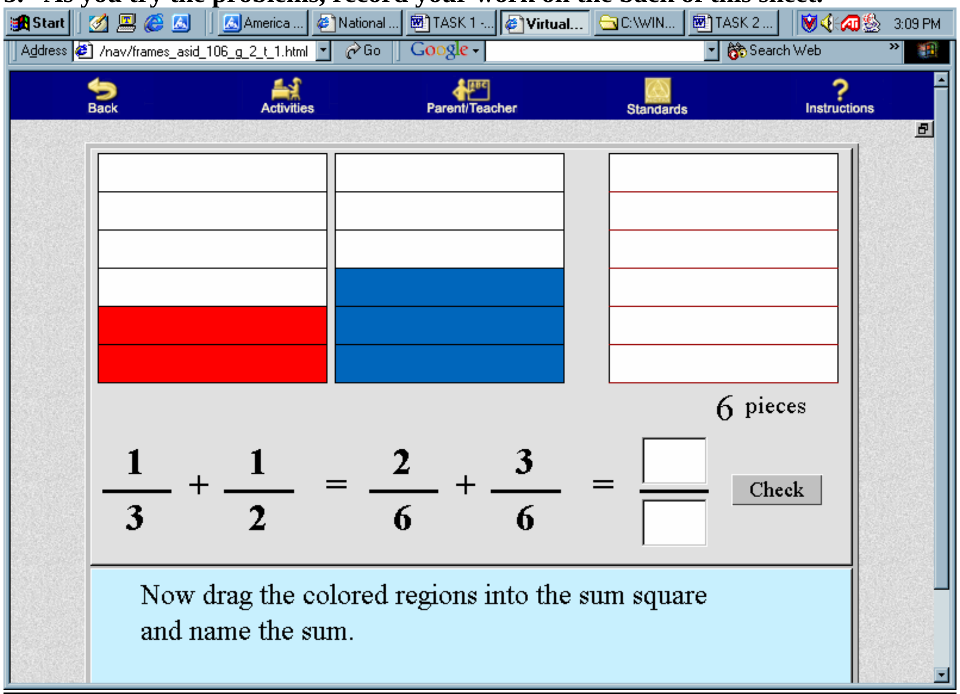
Plus
Minus
Interesting

Go back to the main menu and try FRACTION- Comparing
http://matti.usu.edu/nlvm/nav/vlibrary.html

Fraction- Virtual Manipulative Task Sheet 2

TASK 2: Two Step Sum 

1. **Go to the NLVM at http://matti.usu.edu/nlvm/nav/vlibrary.html**
2. **Click on Number sense grade 3-5**
3. **Click on Addition of fractions.**
4. **Try 5 problems.**
5. **As you try the problems, record your work on the back of this sheet.**



This manipulative illustrates the two-step process of adding proper fractions. The first step in adding fractions is to identify a common group name (denominator). Finding a common group name means separating two (or more) whole groups into the same number of parts.

To complete this activity:

1. Use the arrow keys to separate the whole units into the same number of parts.
2. Enter the appropriate numerator and denominator values for the renamed fractions. Renamed fractions are equivalent. Can you state a rule for renaming?

   The second step is to combine (or add) the renamed fractions.

3. Drag the highlighted parts to form the new graph and type the resulting sum into the fraction box.

4. Click on the "Check" button to see if your answer is correct.

Keep a record of your work in the space below.

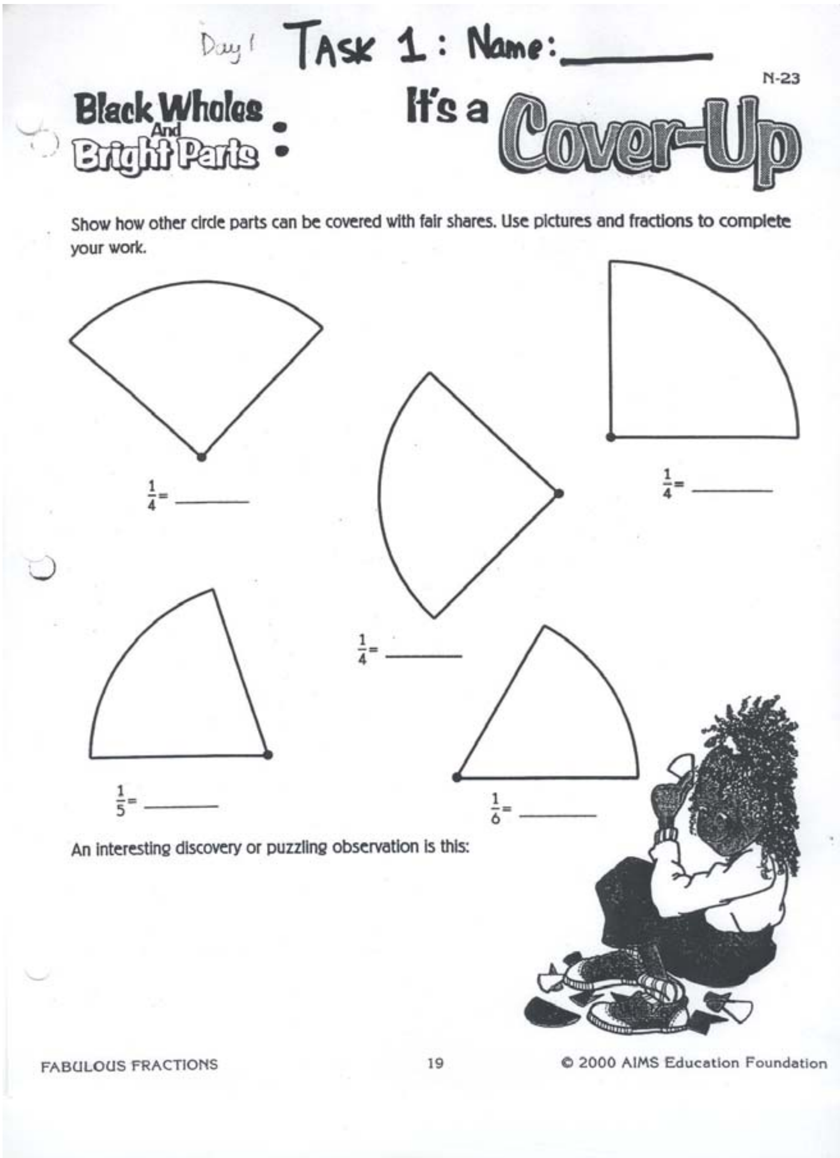
How would you state what you did in YOUR OWN words?

Plus
Minus

Interesting

Go and play the fraction game at
http://standards.nctm.org/document/eexamples/chap5/5.1/index.htm

Fraction-Physical Manipulative Task Sheet 1



Day 1 TASK 1: Name: ________

N-23

Black Wholes And Bright Parts: It's a Cover-Up

Show how other circle parts can be covered with fair shares. Use pictures and fractions to complete your work.

$\frac{1}{4}$ = ________

$\frac{1}{4}$ = ________

$\frac{1}{4}$ = ________

$\frac{1}{5}$ = ________

$\frac{1}{6}$ = ________

An interesting discovery or puzzling observation is this:

FABULOUS FRACTIONS 19 © 2000 AIMS Education Foundation

Fraction- Physical Manipulative Task Sheet 2

TASK 2: Name: ________

N-29

FRACTION CDs

CD Practice

(1)

$\frac{3}{8} =$

$+ \frac{2}{4} =$

______ ______

(2)

$\frac{2}{3} =$

$+ \frac{1}{4} =$

______ ______

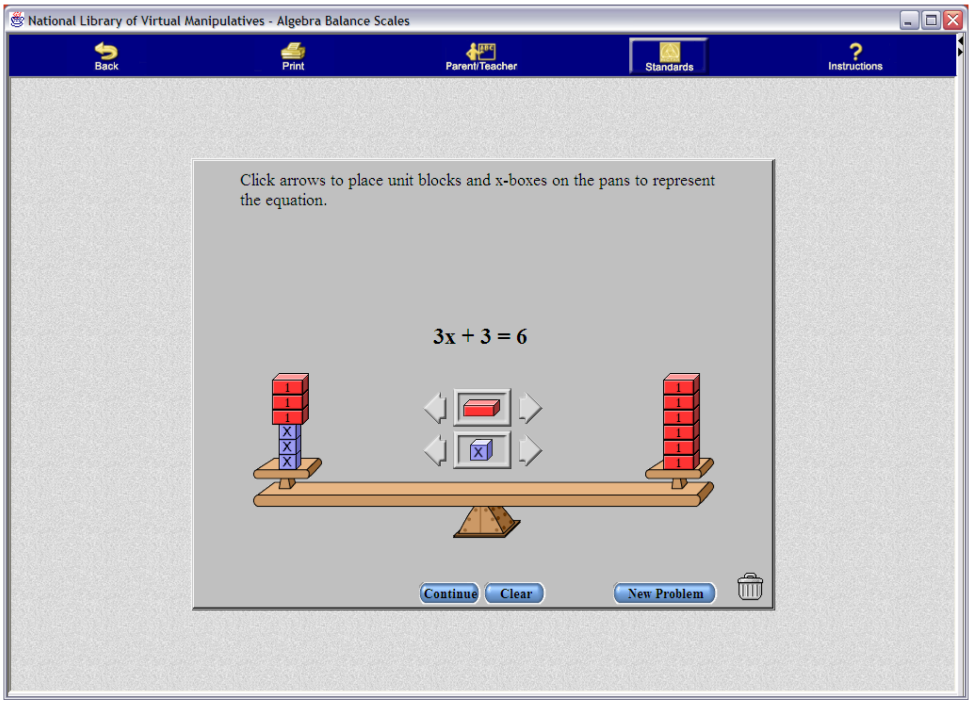
(1)

(2)

Algebra-Virtual Manipulative Task Sheet

Instructions:

This virtual manipulative allows you to solve simple linear equations through the use of a balance beam. Unit blocks (representing 1s) and X-boxes (for the unknown, X), are placed on the pans of a balance beam. Once the beam balances to represent the given linear equation, you can choose to perform any arithmetic operation, as long as you DO THE SAME THING TO BOTH SIDES, thus keeping the beam balanced. The goal, of course, is to get a single X-box on one side, with however many unit blocks needed for balance, thus giving the value of X.



Try several problems and print a record of at least three problems that you solved.
For one of the problem, explain the process of finding the value of x in your own words.

Algebra- Physical Manipulative Task Sheet 1

HANDS-ON EQUATIONS®

Lesson #2
Classwork Sheet

Name: ____________
Grade: ____________

Use your Hands-On Equations Kit to solve:

New Work

1. $2x=x+3$ x= Check: ______

2. $3x=x+4$ x= Check: ______

3. $x+4=2x+3$ x= Check: ______

4. $4x=2x+6$ x= Check: ______

Previous Work

5. ▲▲▲ , [10][2] x= Check: ______

6. ▲▲[1] , [10][5] x= Check: ______

7. ▲[8] , [10][6] x= Check: ______

8. ▲▲[5] , ▲▲▲[1] x= Check: ______

9. ▲▲[1] , ▲[3] x= Check: ______

10. ▲▲[5] , [10][10][5] x= Check: ______

Algebra –Physical Manipulative Task Sheet 2

HANDS-ON EQUATIONS®

Lesson #3 — Name: ____________

Classwork Sheet — Grade: ____________

Use your Hands-On Equations Kit to solve:

New Work

1. 3x+7=4x x= Check: ______
2. x+2+2x=x+10 x= Check: ______
3. x+3x=x+x+10 x= Check: ______
4. 2x+3+3x=x+11 x= Check: ______

Previous Work

5. ▲▲ , [10][2] x= Check: ______
6. ▲▲▲▲ , [10][10] x= Check: ______
7. ▲[1]▲▲ , ▲[5] x= Check: ______
8. 2x+2=x+5 x= Check: ______
9. 3x+1=x+13 x= Check: ______
10. x+3+2x=x+5 x= Check: ______

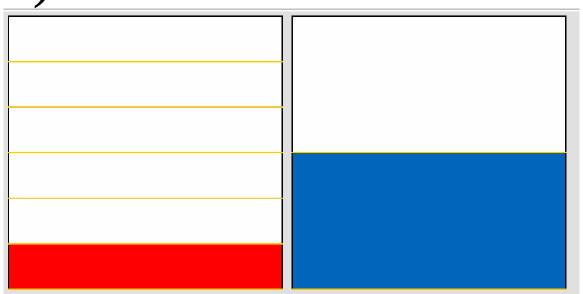
APPENDIX E: PRETEST FOR FRACTION AND ALGEBRA CONCEPTS

Fraction Pretest

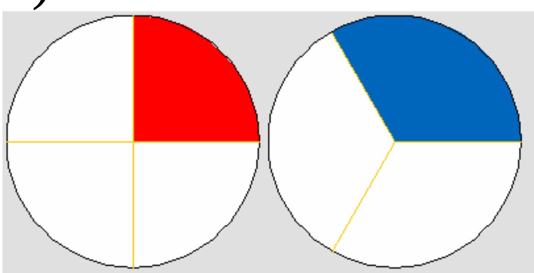
Name: _______________

Add the two fractions.

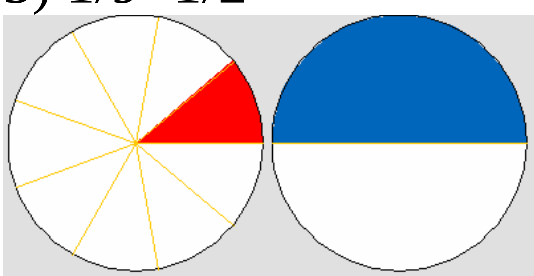
1) 1/6 +1/2=



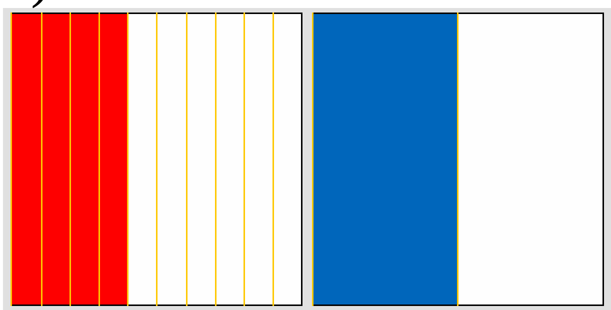
2) 1/4+1/3=



3) 1/9+1/2=



4) 4/10+1/2=



5) **2/4+3/8=**

6) **2/5+3/10=**

7) **2/3+1/4=**

8) **2/9+1/2=**

Solve by drawing a picture, math number sentence and explain how you solved it.

9) Mrs. Reedy used ¼ can of paint for her dining room and 3/8 can of paint for her kitchen. How much paint did she use?

Picture

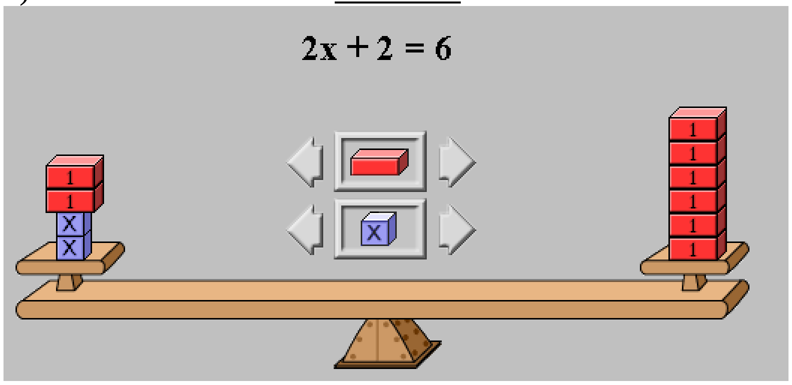
Number sentence

Explanation on how you solve this problem

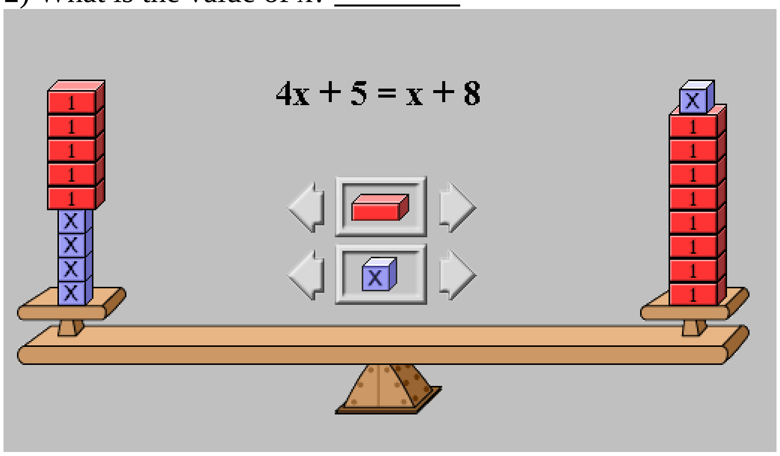
Name: ________________

Algebra Pretest Questions

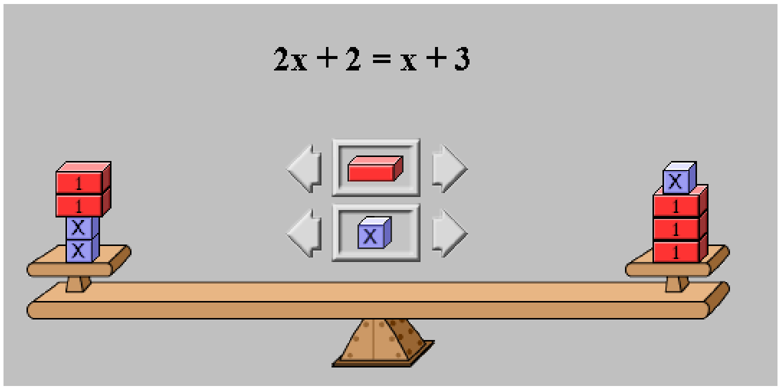
1) What is the value of x? ________



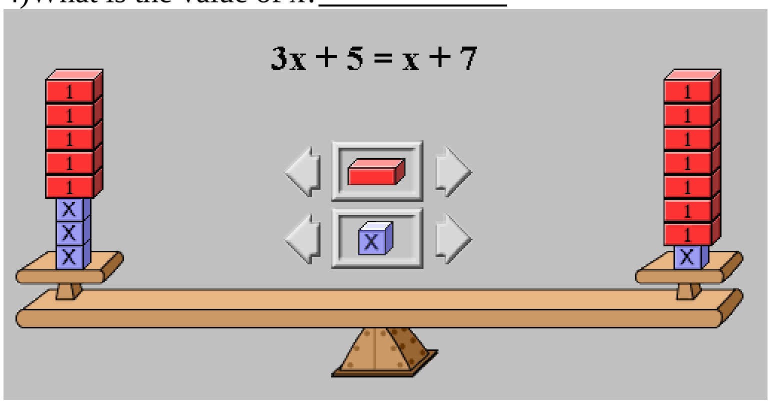
2) What is the value of x? ________



3) What is the value of x?____________



4)What is the value of x?____________



5) $3x+1=7$
x= ______

6) $3x=x+4$
x=______

7) $x+4=2x+3$
x=______

8) $3x+1=x+13$
x=______

Solve by drawing a picture, math number sentence and explain how you solved it.

9) Jeremy has 17 balls. If you have the same number of balls in two boxes and 5 loose balls, how many balls are in the each box? Draw a picture to help you solve this problem and write the algebra sentence that can help you solve this problem.

Picture

Number sentence

Explanation on how you solve this problem.

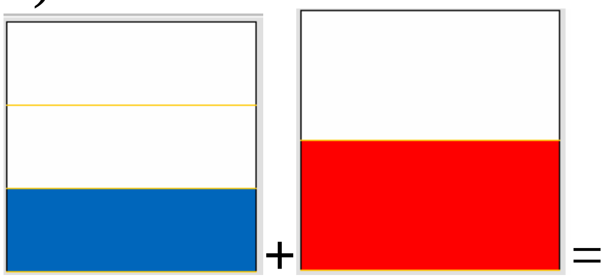
APPENDIX F: FRACTION POSTTEST

Fraction assessment

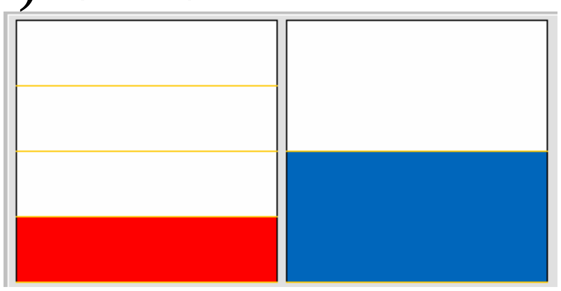
Name: _______________

Add the two fractions.

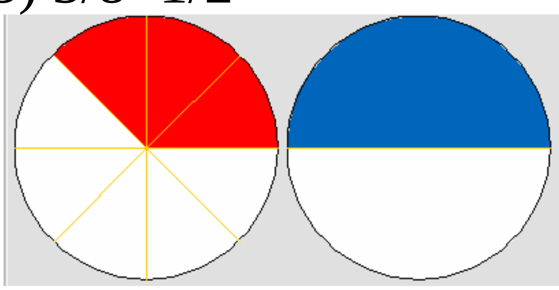
1) 1/3+1/2=



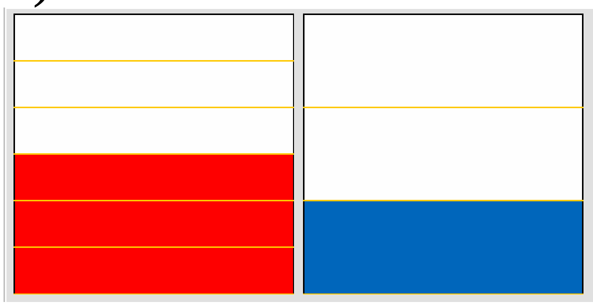
2) 1/4+1/2=



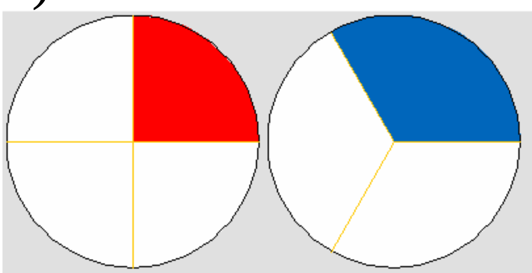
3) 3/8+1/2=



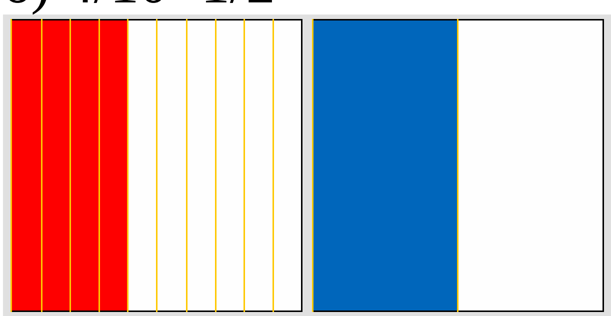
4) 3/6+1/2=



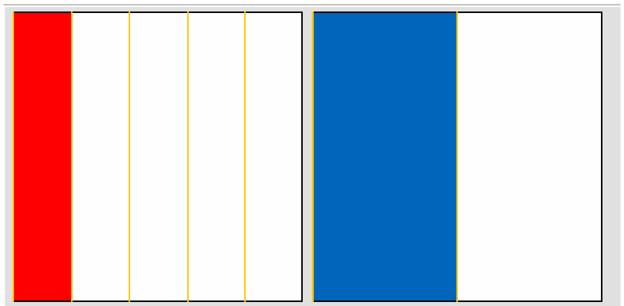
5) 1/4+1/3=



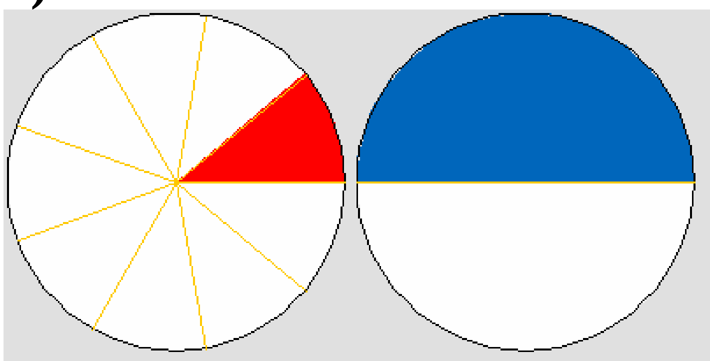
6) 4/10+1/2=



7) 1/5+1/2=



8) 1/9+1/2=



9) 3/4+1/8=

10) 2/3+1/6=

11) 2/4+3/8=

12) 2/5+3/10=

13) 2/3+1/4=

14) 2/9+1/2=

15) 1/3+3/4=

16) ¼+1/5=

<u>Fraction Conceptual Assessment Task.</u>
Draw a picture, write a number sentence and explain how you would solve these two problems.

1) **Mrs. Reedy needs ¼ yard of fabric for the curtains in her office and 3/8 yard of fabric for her table. How much fabric will she need?**

Picture

Number sentence

Explanation on how you solve this problem.

2) Mr. Mahlio bought 1/2 pound of ham and 1/3 pounds of turkey for his sandwich. How much meat did he buy for his big lunch?

Picture

Number sentence

Explanation on how you solve this problem.

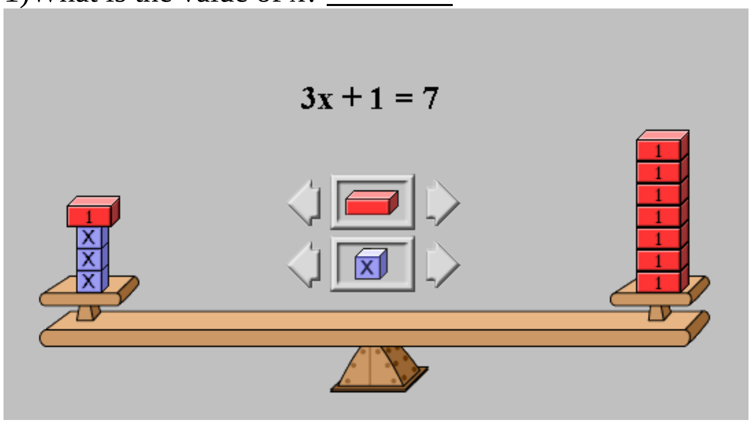
APPENDIX G: ALGEBRA BALANCE EQUATIONS POSTTEST

Algebra balance Equations Assessment

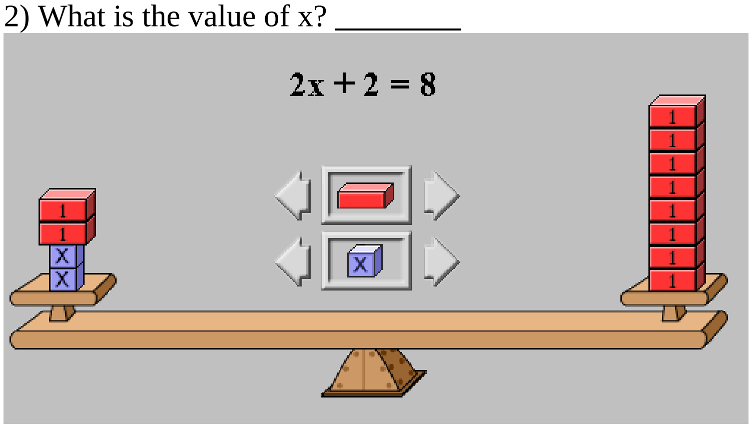
Name: ______________________

Pictorial items:

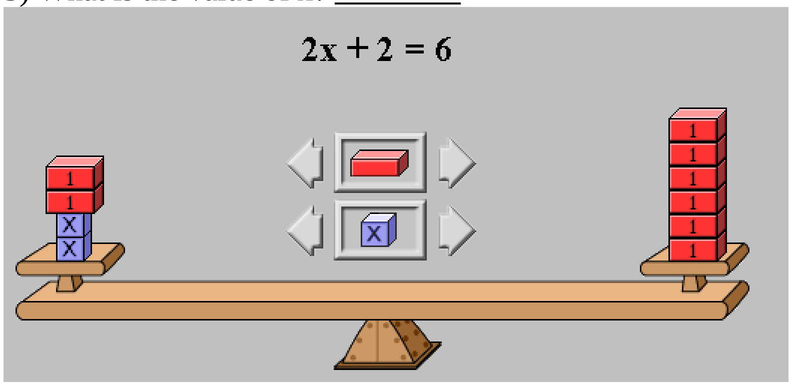
1)What is the value of x? ________



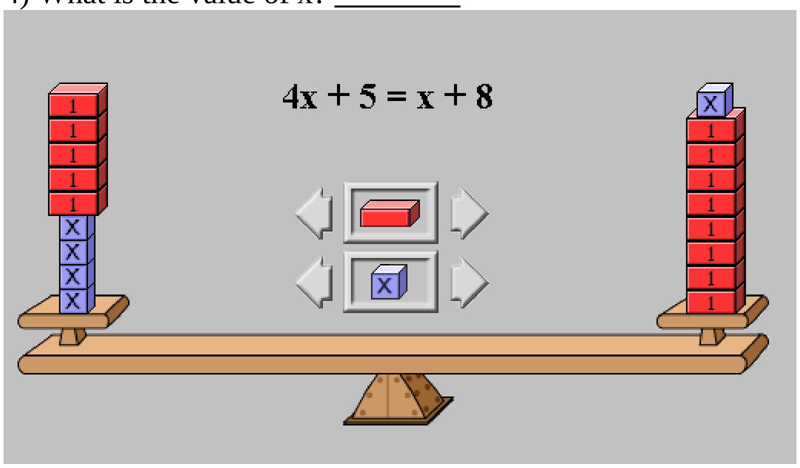
2) What is the value of x? ________



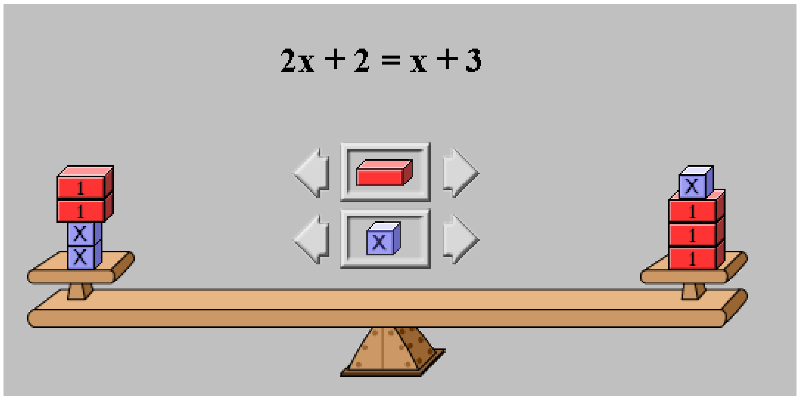
3) What is the value of x? _________



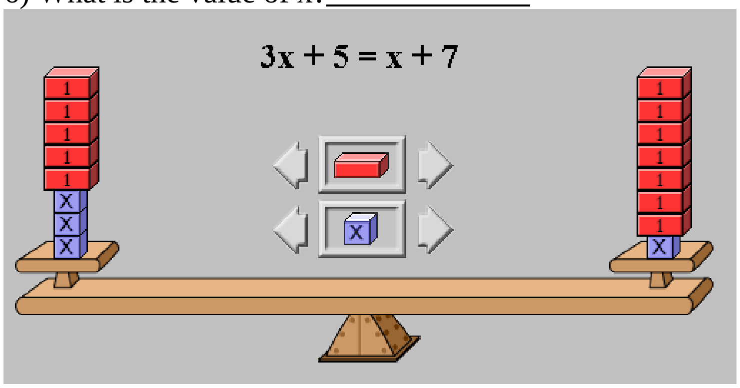
4) What is the value of x? _________



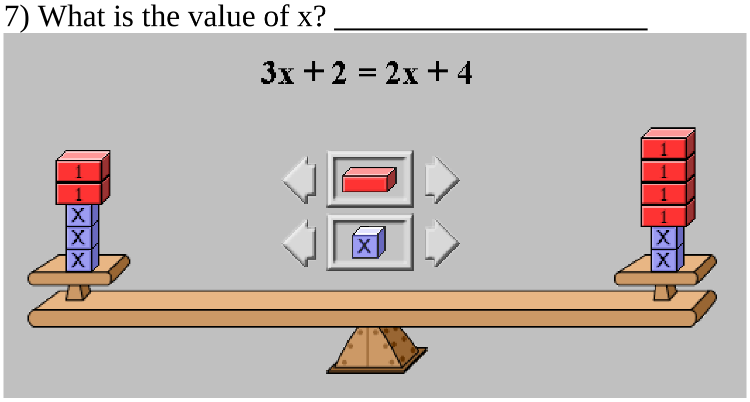
5) What is the value of x?_____________



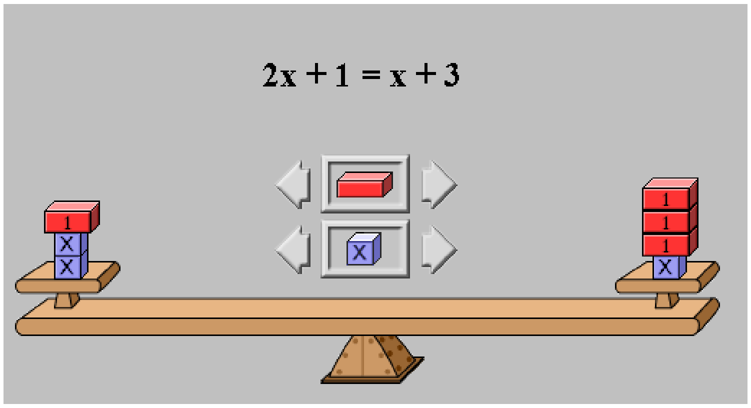
6) What is the value of x?____________



7) What is the value of x? ___________________



8) What is the value of x? ___________________


2x + 1 = x + 3
1
X
X
X
1
1
1
X

9) 2x+2=10
x=______

10) 3x+1=7
x= ______

11) 3x=x+4
x=______

12) x+4=2x+3
x=______

13) 3x+7=4x
x=______

14) x+3x=2x+10
x= ______

15) 3x+1=x+13
x=______

16) 3x+10=5x
x= _______

Conceptual Assessment Task:

Draw a picture to the problems, write an algebra sentence that can help you solve this problem and explain how you solved the problem.

1) You can buy 5 small pizzas for the same price as 3 small pizzas and 10 one-dollar drinks. How much does each pizza cost?

Picture

Number sentence

Explanation on how you solve this problem.

2) Jeremy has 17 erasers. If you have the same number of erasers in 2 boxes of erasers and 5 loose erasers. How many erasers are in the each box? Draw a picture to help you solve this problem and write the algebra sentence that can help you solve this problem.

Picture

Number sentence

Explanation on how you solve this problem.

APPENDIX H: USER SURVEY

Student Survey About Using Manipulatives in Math

Name: ________________________
Circle the Manipulative Type: Virtual or Physical

This week you worked with manipulatives to learn math concepts. Please circle the number that best describes your attitude about this learning tool and give your honest opinions about your experience by answering these questions.

1. Not at all
2. Some
3. A lot

1) Do you like working with these learning tools in math? Please explain.

1 Not at all 2 Some 3 A lot

2) Do these manipulatives help you understand math better? Please explain.

1 Not at all 2 Some 3 A lot

3) Have you ever used manipulatives before? What was your experience like before?
1 Not at all 2 Some 3 A lot

4) I would like to use this tool again to learn other math concepts.

1 Not at all 2 Some 3 A lot

5) I can stay on task easier by using this tool.

1.Not at all 2 Some 3 A lot

6) Using this tool helps me correct my own mistakes.

1.Not at all 2 Some 3 A lot

7) This tool is easy to use.

1.Not at all 2 Some 3 A lot

8) Using this tool becomes boring.

1.Not at all 2 Some 3 A lot

Plus : Were there any features or any special ways that this manipulative helped you learn the math?

Minus:

Interesting:

APPENDIX I: PREFERENCE SURVEY

Preference Survey

Name: ___________________

Read the statements and circle the tool that is more true of each statement.

Statements	Virtual 	Physical 
1. In the future, I would like to use this tool more.	Virtual	Physical
2. Learning with this tool is a good way to spend math time.	Virtual	Physical
3. It is fun to figure out how this learning tool works.	Virtual	Physical
4 Using this tool becomes boring.	Virtual	Physical
5. Working with math problems using this tool is fun like solving a puzzle.	Virtual	Physical
6. I wish I had more time to use these types of tools in math.	Virtual	Physical
7. Learning using this tool is interesting.	Virtual	Physical
8. I can stay on task easier by using this tool.	Virtual	Physical
9. I would feel comfortable working with this learning tool.	Virtual	Physical
10. This learning tool makes me feel uneasy and confused.	Virtual	Physical
11. I can explain how to do math better using this tool.	Virtual	Physical
12. This tool was easy to use.	Virtual	Physical
13. This tool helped me understand work with fraction/ algebra number sentences.	Virtual	Physical
14. This tool helps me get the right answers.	Virtual	Physical

APPENDIX J: INTERVIEW QUESTIONS

Interview questions

1. Did the virtual or physical manipulatives help you learn math? Which form of manipulatives did you like better? Please explain.

2. What features of the manipulatives did you think was helpful for you to understand the math concepts?

3. Did the number sentence on the screen help you while you were working with the virtual manipulatives?

4. Which tool took longer to figure out how to use it?

5. What were the plus, minus, and interesting things about the virtual and the physical manipulatives?

CURRICULUM VITAE

Jennifer M. Suh, an American citizen, was born in Washington, D.C. on April 10, 1971. She was raised in Seoul, Korea and in Northern Virginia. She received a Master of Teaching in Elementary Education (K-8th) with a Bachelor of Art in Psychology in May 1994 at the University of Virginia in Charlottesville, Virginia. She taught two years overseas in Seoul, Korea for the Department of Defense Dependents Schools and earned a Certification in Teaching Elementary Language Immersion Grade 1-8. Following her return to Virginia in 1996, she began a public teaching career in Fairfax County teaching a multiage classroom at Lemon Road Elementary then as a Gifted Education Teacher at Willow Springs Elementary School in Fairfax, Virginia. She also worked as a third grade classroom teacher and a fifth grade mathematics teacher at Little River Elementary in Loudoun County, Virginia.

In 1998, she enrolled in the Ph.D. Education program at George Mason University. She has worked as an adjunct faculty at Marymount University teaching Elementary Mathematics Methods to preservice and inservice teachers. As of Fall of 2004, she has been working as an adjunct faculty at George Mason University teaching mathematics method courses and as a University Facilitator supervising intern teachers at Westlawn Elementary School in Falls Church, Virginia. In addition to teaching, she continues to be involved in presenting research at local, national and international conferences and writing articles for mathematics journals.